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Antenna Spacing Requirement for a Mobile Radio Base-Station Diversity

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The minimum required antenna spacing between two base-station antennas in order to take advantage of spatial diversity technique was investigated. The measurements were made for two cases: (i) the incoming radio signal was perpendicular to the axis of two base-station antennas (the broadside case), and (ii) the signal was in-line with the axis of two base-station antennas (the in-line case). The correlation of signals received from two separated antennas at the base station was found to be much higher for the in-line case than for the broadside case with any given antenna spacing. For correlation up to 0.7, from which most of the advantage of two-branch diversity can still be obtained, we found the minimum required antenna spacing is around 70λ–80λ for the in-line case and 15λ–20λ for the broadside case. In order to achieve a correlation always less than 0.7 between two base-station signals regardless of the arrival direction of the incoming signal, a triangular configuration with a three-antenna array used with a three-branch diversity receiver is proposed, requiring less antenna spacing in the array than for a two-antenna setup.

I. INTRODUCTION

It has been shown previously that diversity reception techniques at the mobile unit often help to reduce the fading rate of a mobile radio signal.[1–3] Here we try to determine the limitations on using space diversity at the base station in order to reduce the signal fading. In particular, we would like to know how large the antenna spacing should be between two base-station antennas in order to take advantage of the diversity technique. If this turns out to be practical, then we might prefer to build a diversity system, even a complicated one, from the economical point of view at the base station, and let the transmitter and receiver in the mobile unit be as simple as possible.[4,5] In the experiments we varied the antenna spacing between two base-station antennas and calculated the cross-correlation of the envelopes of the signals received from these antennas for a number of different antenna spacings. We also determined the theoretical relationship between the cross-correlation and the cumulative distribution curve of the combined signal. If we select a particular cumulative distribution curve as acceptable for diversity operation, the corresponding correlation then indicates the required antenna spacing.

Using the experimental correlation data of both the in-line propagation case and the broadside propagation case as a guide, we will try to reduce the required antenna spacings by using an array of three base-station antennas.

II. EXPERIMENTS

A mobile radio transmitter was set up at 836 MHz in a station wagon with a $\lambda/2$ dipole mounted vertically. This transmitted to two receiving horns having 24 degrees beam width in the horizontal plane and located at the north end of Crawford Hill in Holmdel, New Jersey. During the measurements the station wagon was driven at a constant speed of 15 mph on three selected streets in the Keyport area, as shown in Fig. 1a and b. The horn antennas were used to reduce the local scattering at the base, thus simulating the conditions at a typical installation where the basic antenna is mounted well above nearby objects. Two of the streets chosen were in line with the radius vector of the base station, and the other was perpendicular to the radius vector. (The symbols •→ and ←• in Fig. 1a and b indicate different runs on the same streets. The dot indicates that the data have been received around this point on the street, and the arrow indicates the motion of the transmitter.) Figure 1a shows the experimental set up for the case where the incoming radio signal is perpendicular to the axis of two base-station antennas (the broadside case). In this case, the S → N runs were closer to the base station than the N → S runs on Main Street and on Broadway, as shown in Fig. 1a. The distance from the mobile transmitter to the base-station receivers was about three miles. The separation between the two base-station receiving antennas was variable from 25λ to 70λ in 10λ steps starting from 30λ, i.e., 25λ, 30λ, 40λ, $\cdots$.

Figure 1b shows the experimental setup for the case where the incoming radio signal is parallel to the axis of two base-station antennas (the in-line case). In this case, the S → N runs and N → S runs were made approximately in the same sections of the streets. The distance

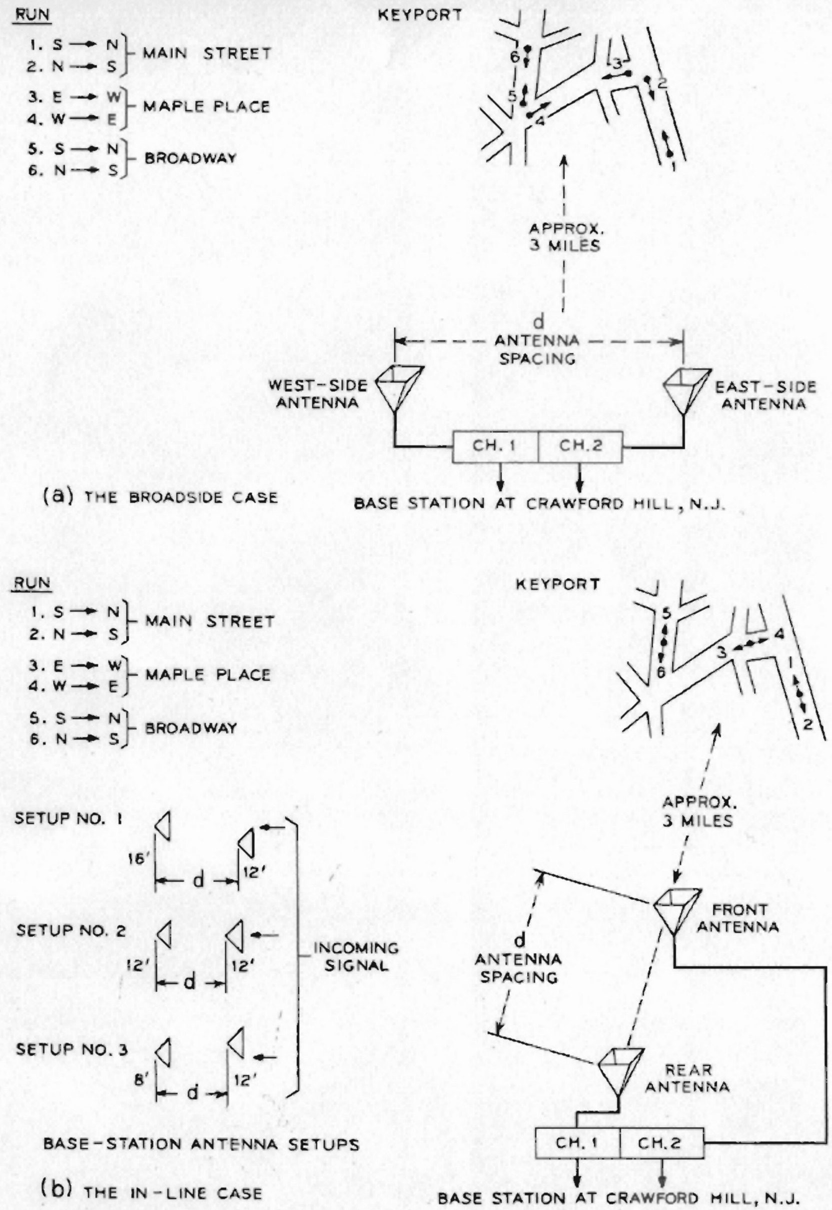

Fig. 1a—Map of testing area (Keyport, N. J.), the broadside case.

Fig. 1b—Map of testing area (Keyport, N. J.), the in-line case.

from mobile transmitter to the base-station receivers was also about three miles. The separation between the two base-station receiving antennas was variable from 30λ to 100λ in 10λ steps. The height of the front base-station antenna was fixed at 12 feet. The height of the rear antenna was variable in three steps, 16 feet, 12 feet, and 8 feet.

For both cases, each signal received from each antenna was fed to a separate receiver, which was calibrated from a common source. The two signals were recorded simultaneously on a magnetic tape recorder.

III. EXPERIMENTAL CROSS-CORRELATIONS

All data obtained from the two-channel base-station receiver were digitized at a 500-Hz sampling rate, which is fast enough to sample the data adequately. There are several questions which need to be answered before calculating the cross-correlation of two received signals. First, are the mean values of a signal in different time intervals (local means) different, and, if so, how should we compensate for this? Second, how many sample points should be taken for each of two received signals to calculate their cross-correlation? Third, are the processes of the signals we received stationary in the wide sense?[6] These can tell us that the correlations we have obtained are independent of time. Statistically analyzing our data, we have had to justify several statistical properties of individual pieces of data before calculating the cross-correlations among them:

(*i*) *Treatment of Local Means*–We have found that the mean values of a signal in different time intervals (local means) are different. The variations in the local means of a received signal are due to the different contours of land between the transmitter and the receiver at different time intervals. These variations affect the auto-correlation function of a received signal. Hence, we normalized the local means of the received signal, before calculating its auto-correlation, as follows: We broke a piece of data into many time intervals, then each sample was divided by the mean of the signal in that time interval. In our data, we used a time interval a half-second long, i.e., 250 samples.

(*ii*) *The Number of Sample Points to be Taken*–We have found from the auto-correlation curves of most data that their first nulls occur at a delay, τ, of 20 samples. Then we may say that there is no correlation between two pieces of data separated by a delay of 20 samples. Since these pieces of data are Rayleigh distributed,[4] if they are uncorrelated, they are also independent. Now we look at this a different way: how long will a piece of data be which can from an ensemble of , say, 100 independent subsets of data? Since every subset of data with a delay of at least 20 samples from each other is an independent subset, a piece of data containing over 2000 samples is required to provide an ensemble of 100 independent subsets. We know that 100 independent subsets of data are enough to calculate the ensemble average. Therefore, from the concept of ergodicity, we need a piece of data containing over 2000 samples in order to meet the requirement "time average equals ensemble average". However, most of the local means of all runs vary rapidly after 2500 samples. Therefore, we chose 2500 sample points as a unit piece of data.

(*iii*) *Verification of Stationarity*–If a piece of data is stationary, the auto-correlation of a piece of data $x(t)$ should be independent of time t. This means that if we pick any arbitrary starting point, say one second after the first sample point, and calculate the auto-correlation, the result should be the same as if we had chosen the first sample point as the starting point. This evidence was observed in our data. Hence we have shown that our data are stationary.

In summary, we have found that the local means of two signals should be properly factored out, and that 2500 sample points are sufficient for calculating the cross-correlation. Under these two requirements, the processes of the signals we received were verified to be stationary in the wide sense.

Assuming two signals received from two base-station antennas, separated by distance d, are $x(t)$ and $y(t)$, the cross-correlation coefficient of $x(t)$ and $y(t)$ delayed by time τ can be expressed as

$$\rho(t, \tau, d) = \frac{\overline{x(t)y(t+\tau)} - m_x m_y}{\sqrt{\overline{x^2(t)} - m_x^2}\sqrt{\overline{y^2(t+\tau)} - m_y^2} \qquad (1)$$

where m_x and m_y are mean values of $x(t)$ and $y(t+\tau)$, respectively. Since we have shown that our data are stationary, and also we let $\tau = 0$, equation (1) becomes

$$\rho(d) = \frac{\overline{x(0)y(0)} - m_x m_y}{\sqrt{\overline{x^2(0)} - m_x^2}\sqrt{\overline{y^2(0)} - m_y^2}}. \qquad (2)$$

After normalizing the local means of the two signals from the two channels (receivers), we used 2500 sample points and calculated the cross-correlation as a function of antenna spacing for two cases.

3.1 *The Broadside Case*

The cross-correlation as a function of base-station antenna spacing was determined for runs on three streets (Main Street on Fig. 2, Maple

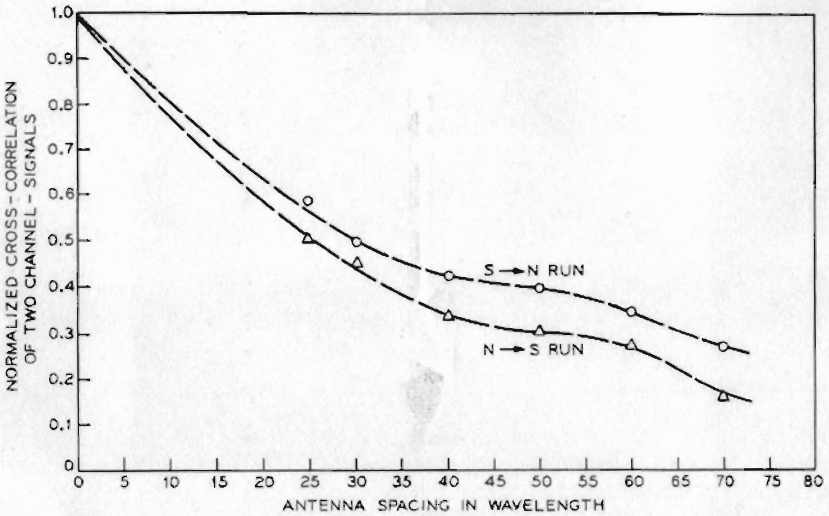

Fig. 2—The correlation coefficient of two base-station antennas broadside with the incoming wave vs antenna spacing for runs on Main St., Keyport, N. J.

Place on Fig. 3, and Broadway on Fig. 4). From these three figures, there are several major points to be disclosed:

(*i*) The cross-correlation decreases as the base-station antenna spacing increases, as one would expect.

(*ii*) In general, the cross-correlation is somewhat higher when the mobile transmitter is nearer the base station, such as the S → N runs shown in Figs. 2 and 4.

(*iii*) The tops of the trees at the northwest boundary of Crawford Hill were high and thus partially scattered the incoming signal from the northwest direction. Hence, the signals received from Broadway and the west side of Maple Place were lower. The cross-correlations obtained from these locations were also low, due to this local scattering.

(*iv*) To measure the cross-correlation for an antenna spacing of 70λ, we moved the west-side receiving antenna farther left by 10λ. Therefore, the tops of trees more affected the incoming signal received by Channel 1 from Broadway. Hence, the two points for this antenna spacing shown on Fig. 4 have very low values.

(*v*) The highest cross-correlation for the six runs for an antenna spacing of 25λ was 0.65, and for an antenna spacing of 70λ was about 0.27.

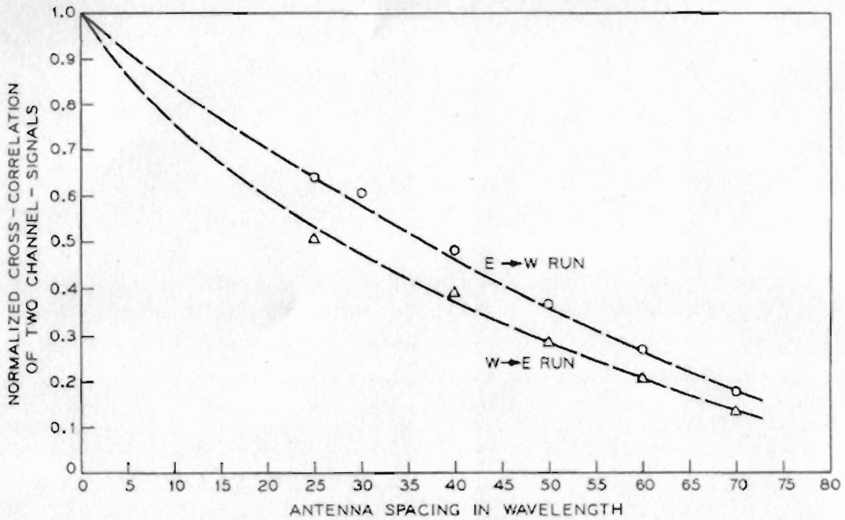

Fig. 3—The correlation coefficient of two base-station antennas broadside with the incoming wave vs antenna spacing for runs on Maple Place, Keyport, N. J.

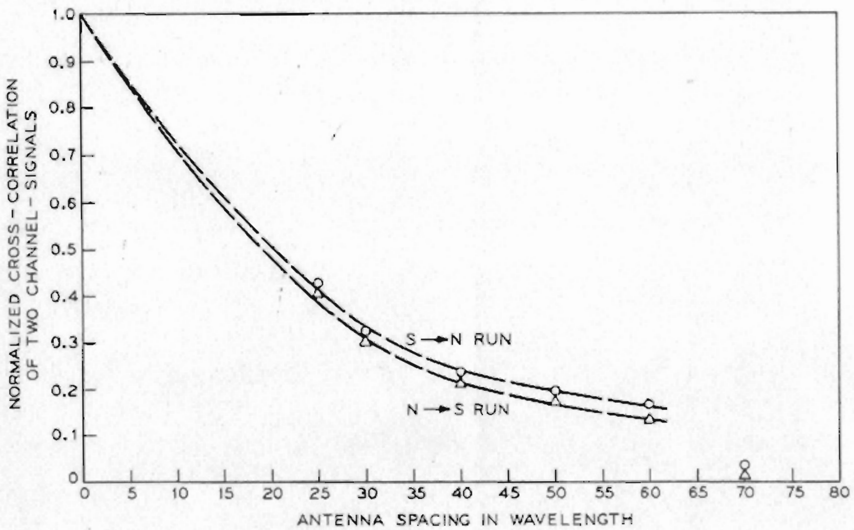

Fig. 4—The correlation coefficient of two base-station antennas broadside with the incoming wave vs antenna spacing for runs on Broadway, Keyport, N. J.

3.2 *The In-Line Case*

The cross-correlation as a function of base-station antenna spacing was determined for runs on three streets (Main Street, Maple Place, and Broadway) for three values of base-station antenna height (see Fig. 1b), and the results shown in Figs. 5 through 13. Each figure shows the correlation for a particular antenna height for runs on a particular street. Assuming that the correlation at zero antenna spacing is one, we can draw a curve for the best fit to our measured correlation points for spacing from 0 to 100λ for each run. From these nine figures, there are several major points to be disclosed:

(*i*) Between the broadside case and the in-line case: The correlation coefficient of the in-line propagation case is much higher than that of the broadside propagation case for a given antenna spacing. For antenna spacing = 70λ, the correlation coefficient is about 0.2 for the broadside propagation case and about 0.7 for the in-line propagation case. Alternatively, to get a correlation of 0.7, the antenna spacing merely needs to be 25λ in the broadside case but 70λ in the in-line case.

(*ii*) Direction of runs: From Figs. 5–7 (Main Street) and Figs. 11–13 (Broadway), we find that the correlations are higher for those runs (S → N) when the mobile radio transmitter was traveling away from the base station than those (N → S) when traveling toward the base station. Figures 8–10 (Maple Place), for runs perpendicular to

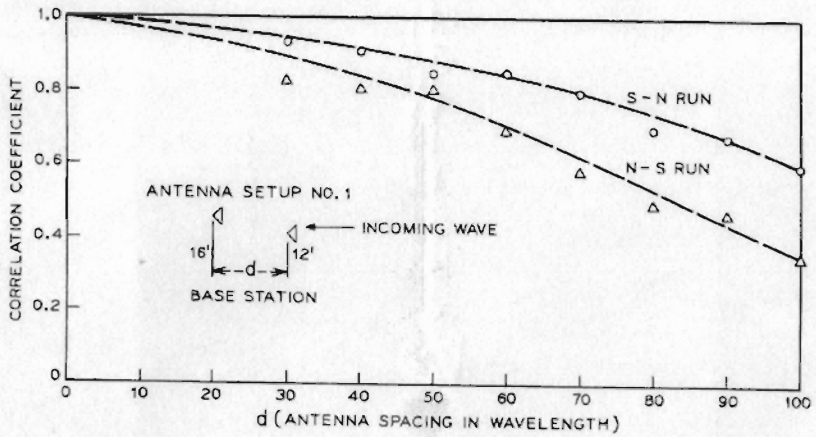

Fig. 5—The correlation coefficient of two base-station antennas (setup no. 1) in line with the incoming wave vs antenna spacing for runs on Main St., Keyport, N. J.

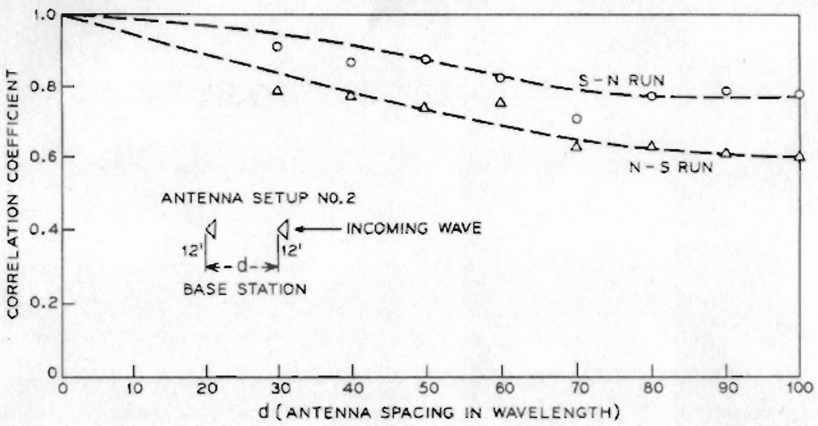

Fig. 6—The correlation coefficient of two base-station antennas (setup no. 2) in line with the incoming wave vs antenna spacing for runs on Main St., Keyport, N. J.

the radius vector of propagation, show that the signal correlations from runs in different directions, E → W or W → E, are not noticeably different.

(*iii*) Antenna height: The variation of height of the base-station antennas apparently has minor effects on the correlations of the received signals. (Figs. 5–13).

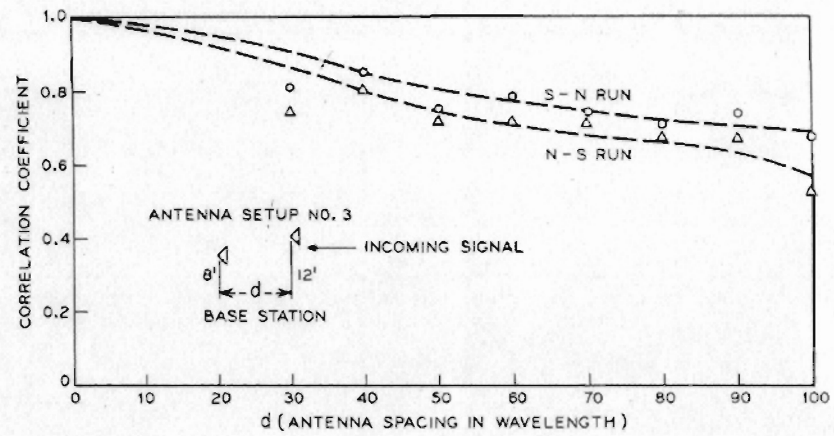

Fig. 7—The correlation coefficient of two base-station antennas (setup no. 3) in line with the incoming wave vs antenna spacing for runs on Main St., Keyport, N. J.

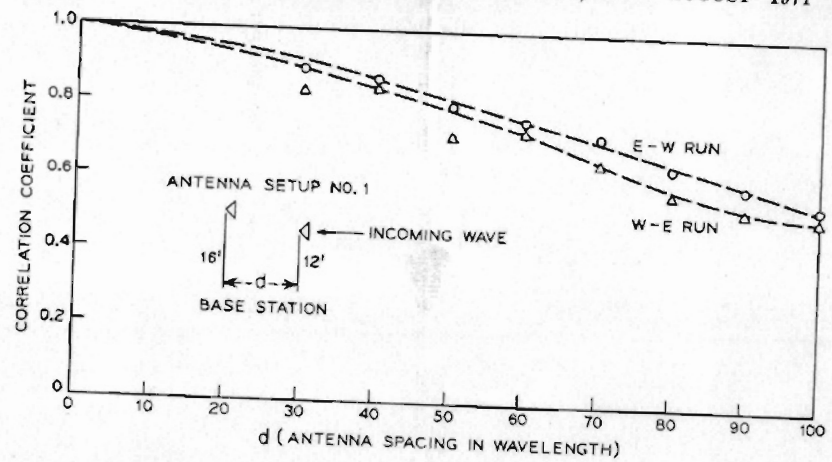

Fig. 8—The correlation coefficient of two base-station antennas (setup no. 1) in line with the incoming wave vs antenna spacing for runs on Maple Place, Keyport, N. J.

(*iv*) Different streets: On Main Street the buildings are taller and spaced closer together than on the other two streets. The buildings on Maple Place are again taller and spaced closer than those on Broadway. This may be the reason that the average correlation drops slightly faster on Main Street and more slowly on Broadway as the antenna spacing increases.

(*v*) Local scattering at the base station: Most of the correlation

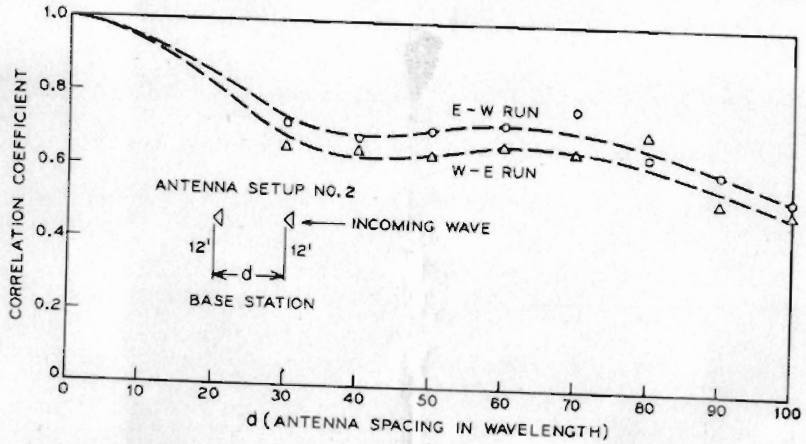

Fig. 9—The correlation coefficient of two base-station antennas (setup no. 2) in line with the incoming wave vs antenna spacing for runs on Maple Place, Keyport, N. J.

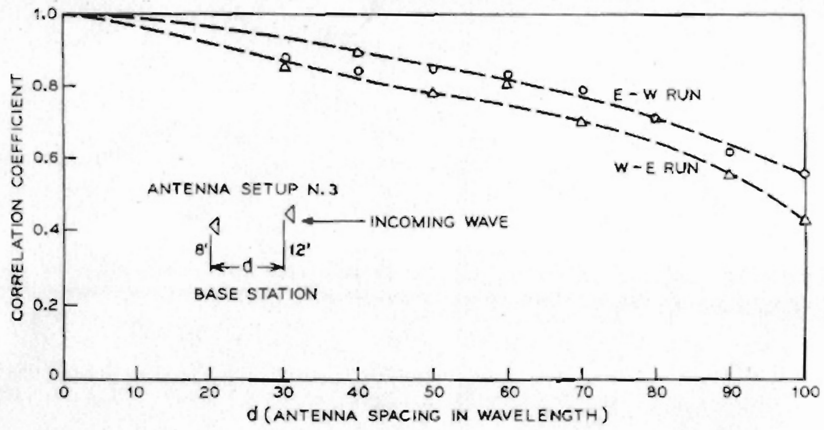

Fig. 10—The correlation coefficient of two base-station antennas (setup no. 3) in line with the incoming wave vs antenna spacing for runs on Maple Place, Keyport, N. J.

curves in Figs. 5–13, are monotonically decreasing as the antenna spacing increases. A few curves have dips; for example, see Fig. 12. These are perhaps caused by the local scattering at the base station.

(*vi*) Upper bound of the correlation data: Figure 14 shows the upper bound of the correlation data for the three antenna setups versus the antenna spacing d. When $d = 70\lambda$, the highest correlation is 0.85.

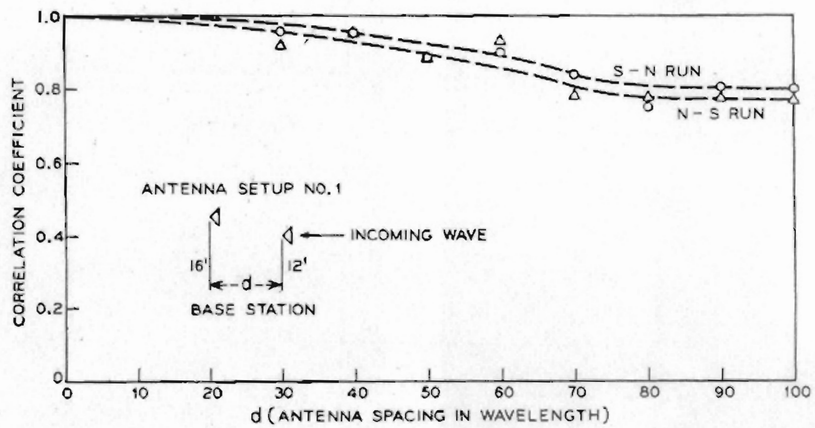

Fig. 11—The correlation coefficient of two base-station antennas (setup no. 1) in line with the incoming wave vs antenna spacing for runs on Broadway, Keyport, N. J.

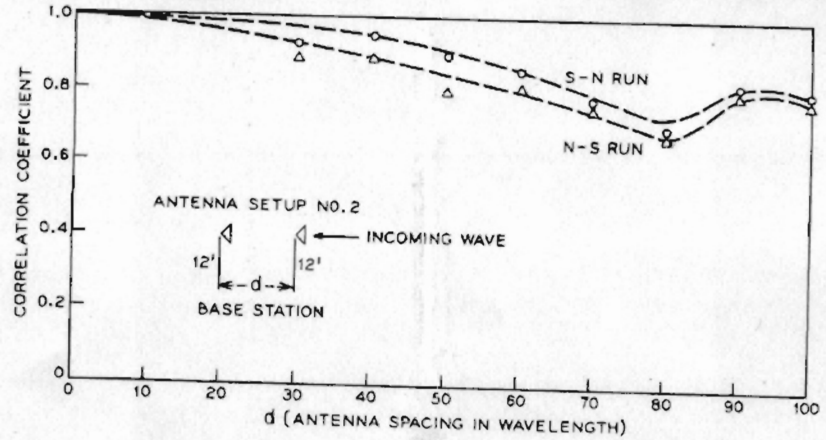

Fig. 12—The correlation coefficient of two base-station antennas (setup no. 2) in line with the incoming wave vs antenna spacing for runs on Broadway, Keyport, N. J.

IV. EFFECT OF CORRELATION ON DIVERSITY

Having obtained the cross-correlation as a function of antenna spacing, we would now like to ask how great a cross-correlation coefficient between two received signals we can tolerate and still realize a signal improvement in a two-branch predetection diversity receiver.

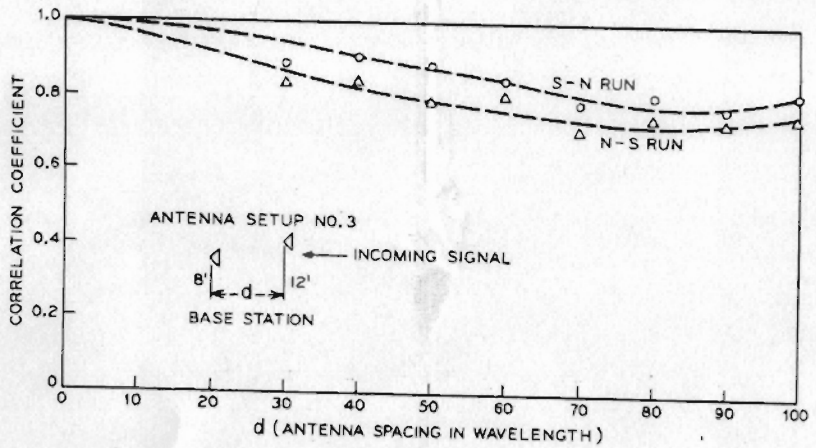

Fig. 13—The correlation coefficient of two base-station antennas (setup no. 3) in line with the incoming wave vs antenna spacing for runs on Broadway, Keyport, N. J.

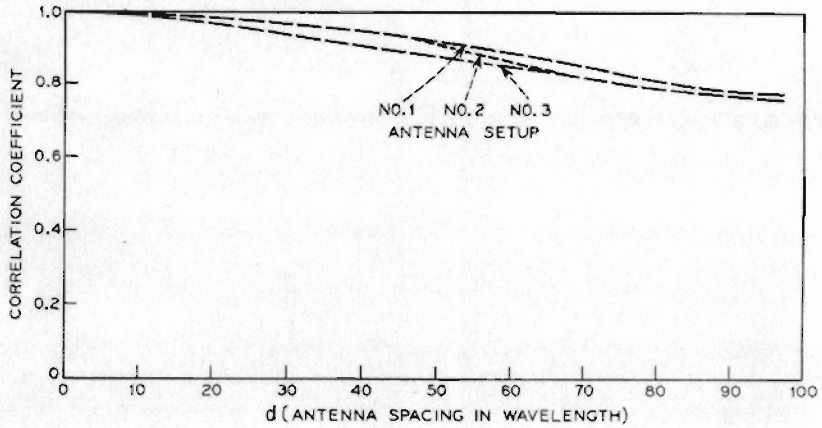

Fig. 14—Upper bound of correlation coefficients from different antenna setups at the base station (in-line case) with a mobile radio transmitting from three different streets in Keyport, N. J.

To do this we will find the cumulative distribution of a two-branch diversity combined signal as a function of the cross-correlation of the two incoming signals.

Assume that the two signals received by a two-branch diversity receiver are complex gaussian random variables[4] in a short-term fading (a piece of data a few hundred wavelengths long). If these two signals are independent, i.e., no cross-correlation, the cumulative distribution of a two-branch maximum ratio diversity* signal can be expressed as[7]

$$P(\gamma < x) = 1 - \left(1 + \frac{x}{\Gamma}\right) \exp\left(-\frac{x}{\Gamma}\right) \tag{3}$$

where γ is the sum of the instantaneous SNR's on the individual branches, Γ is the average SNR of a single channel, and x is a value greater than γ. For the case that the two received signals are not independent, we can obtain the cumulative distribution from Ref. 9 by inserting equation (10-10-21) into equation (10-10-22) for a two-branch maximum-ratio diversity. After some manipulation, we get

* The cumulative distributions of a two-branch maximum-ratio diversity combining signal and a two-branch equal-gain diversity combining signal are very similar, differing only by 0.49 dB[8] if the two branches are independent.

$$P(\gamma < x) = 1 - \left\{ \frac{(1+\sqrt{\rho}) \exp\left[-\frac{x}{(1+\sqrt{\rho})\Gamma}\right] - (1-\sqrt{\rho}) \exp\left[-\frac{x}{(1-\sqrt{\rho})\Gamma}\right]}{2\sqrt{\rho}} \right\} \tag{4}$$

where γ, x, and Γ are as previously described and ρ is the cross-correlation coefficient between the envelopes of two-branch signals. Equations (3) and (4) are plotted in Fig. 15. The abscissa is the SNR with respect to the mean value of the SNR of a single channel, in dB. We see that the curve for $\rho = 0.7$ is very close to that for $\rho = 0$. This means that for correlations between the two branches up to 0.7, the advantage of using diversity technique is still good. Figure 15 also shows that 99.9 percent of the time the SNR for $\rho = 0$ is above -13.3 dB, while at the same percentage, the SNR for $\rho = 0.7$ is above -15.8 dB. The difference is 2.5 dB. Comparing the two-branch signal for $\rho = 0$ with a single-channel signal at the 99.9 percent level, the difference is 17 dB. For a probability of 99.9 percent, the difference between two two-branch signals for $\rho = 0$ and $\rho = 0.7$ is still 2.5 dB; however, the difference between a two-branch signal with $\rho = 0$ and a single-channel signal is 22 dB. Hence, we could say that, if a two-branch signal for $\rho = 0.7$ is taken, the improvement over a single channel at the 99.99 percent level is greater than at the 99.9 percent level. Figure 16 gives a clear view of the two-branch diversity improvement over a single-channel signal for different correlations. The improvement becomes less as the correlation increases. Also the improvement becomes greater at the higher percentage signal level than at the lower percentage signal level, as mentioned previously. For ρ between 0 and 0.7, the diversity advantage changes very little. Hence, we may say that $\rho = 0.7$ is a reasonable value to pick for the maximum cross-correlation which can be tolerated in order to take good advantage of the diversity technique. The antenna spacing for a cross-correlation of 0.7 is around 15λ–25λ for the broadside case (see Figs. 2–4) and is 70λ–80λ for the in-line case (see Figs. 5–13).

V. CONSIDERATION OF A THREE-BRANCH DIVERSITY SYSTEM

As we have seen, to get a correlation of 0.7 between two base-station signals in the in-line case requires antenna spacings of 70λ to 80λ, and in the broadside case requires antenna spacings of 15λ to

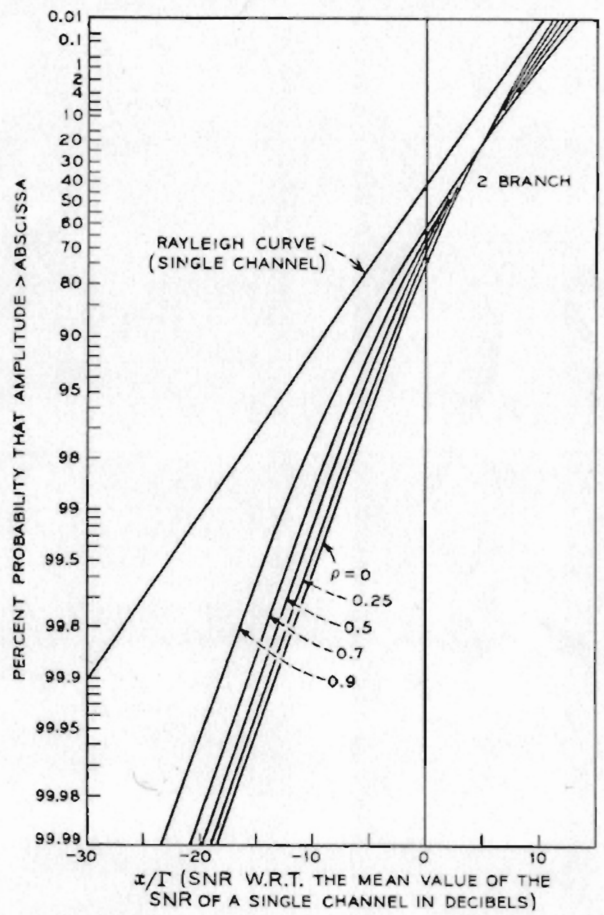

Fig. 15—The cumulative distribution curves vs correlation for a two-branch maximum-ratio diversity receiver.

25λ. Hence the broadside case is better than the in-line case as far as saving the antenna space is concerned. Since the base-station antennas are set up not only for one mobile unit in one particular direction but rather for all the mobile units under its radio coverage, some mobile units may well be in the in-line case to the base station if only two base-station antennas are considered. If we try to reduce the antenna spacing and still meet the same correlation requirement of 0.7 or less regardless of the direction of the incoming signal arrival, a triangular antenna array may provide a solution. In a triangular

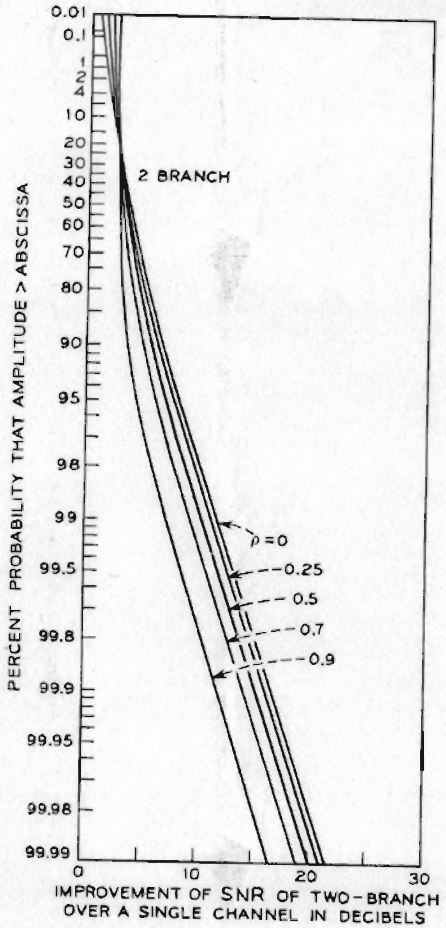

Fig. 16—Improvement of signal-to-noise ratio of a two-branch signal over a single-channel signal.

array, we have three cross-correlations between three antennas. When an incoming mobile radio signal is in line with two antennas of the array, then the correlation with these two antennas is very high but the other two correlations are lower, since they are more nearly broadside to the incoming signals. Because two of the three antennas are always approximately broadside to an incoming signal in a triangular array, we may always get at least the advantage of two-branch diversity but never more than three-branch.[10] Furthermore, by making more correlation measurements for arrival angles between in-line and broadside cases in the future, the exact performance of the triangular array could be calculated.

VI. CONCLUSION

The measurements reported here show that the correlation of the in-line propagation case is much higher than that of the broadside propagation case for any given antenna spacing. To get a correlation of 0.7, the antenna spacing merely needs to be 25λ in the broadside case but 70λ–80λ in the in-line case. Direction of runs and variation of height of the base-station antennas have minor or insignificant effects on the correlation. Local scattering at the base station may not have been entirely eliminated in our test; this, if present, would reduce the apparent correlations. In this case, larger separations might be required in actual situations, depending on the amount of local scattering that existed. Further work is needed to resolve this point.

The theoretical analysis has pointed out that the advantage of a two-branch diversity can be obtained when the cross-correlation between branches is less than 0.7.

In order to achieve a condition on the cumulative distribution curve equivalent to a correlation always less than 0.7 between two base-station antennas regardless of the direction of the incoming signal arrival, yet with antenna spacing less than the in-line case, a triangular configuration with a three-antenna array used with a three-branch diversity receiver is proposed. We estimate that, in a triangular array, the antenna spacing between any two of three antennas could be around 40λ for its cumulative curve to be better than that of a two-branch receiver with correlation of 0.7 regardless of the direction of the incoming signal onto the triangular array. The idea of applying the diversity scheme at the base station could then be realized.

VII. ACKNOWLEDGMENTS

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A Simple Interframe Coder For Video Telephony

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The technique of exchanging resolution according to the amount of movement in a picture has been previously described; in stationary parts of the picture the temporal resolution is reduced while in moving parts of the picture the spatial resolution is reduced. Here, we describe a method of applying resolution exchange to a differentially quantized (DPCM) signal. The resulting channel capacity required for the subjectively satisfactory transmission of the differential signal is halved. The coder is simpler than most interframe coders and should not increase the sensitivity of the system to channel errors.

I. INTRODUCTION

In a previous paper we described a way to halve the channel capacity required for the subjectively satisfactory transmission of an 8-bit PCM television signal by exchanging spatial and temporal resolution according to the amount of movement in the local part of the picture.[1] Every second picture element ("pel") is sampled and in stationary areas of the picture the values of the unsampled pels are interpolated from adjacent temporal samples (reduced temporal resolution); in the moving areas the values of the unsampled elements are interpolated from neighboring sampled elements in the same line (reduced spatial resolution).

We would like to apply this technique to a signal whose bit rate has already been reduced by an element-to-element differential quantizer (EDQ), e.g., the *Picturephone*® codec. Unfortunately, halving the horizontal sampling rate, as in Ref. 1, increases the amplitude of the sample-to-sample difference signal which, in turn, requires a larger number of quantizing levels for adequate representation. There are two ways around this problem: