Multi-Channel RF Measurements

Bob Cutler
Senior Member, Technical Staff
Signal Analysis Division

TITLE: Multi-Channel RF Measurements

ABSTRACT: Smart-antenna systems will be used to demonstrate the application of multi-channel RF signal analysis to modern digital communication systems. Instruments with more than one input channel are not new. Scopes and network analyzers are obvious examples. Spectrum and vector signal analyzers, however, have generally been limited to a single RF analysis channel. This paper explores applications and measurement techniques that can be applied when two phase-coherent, time-aligned measurement channels are available. For those unfamiliar with the technology, the presentation includes a brief introduction to smart-antenna technology.

Topics

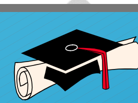
- •Introduction to Smart Antennas
- •Basic statistics, noise
- •Cross-channel measurements and metrics
- •Multi-channel applications and measurements

In this paper, we will start with a short introduction to smart antenna technology. Many of the problems faced by designers of smart antenna systems can be more easily solved using measuring equipment with multiple coherent-channel capability.

Multi-channel measurements allow significantly more than twice the analysis of two one channel instruments. For example, with two single channel analyzers, you can measure two spectrums. With two coherent channels, you can measure two single channel spectrums, plus the cross-channel spectrum. To take full advantage of this new capability, some basic understanding of noise and statistics is required, so a short review of this topic is also included.

Finally, we will give some practical applications for multi-channel measurements.

What Makes an Antenna Smart?



"A smart antenna system combines multiple antenna elements with a signal-processing capability to transmit and receive in an adaptive, spatially sensitive manner"

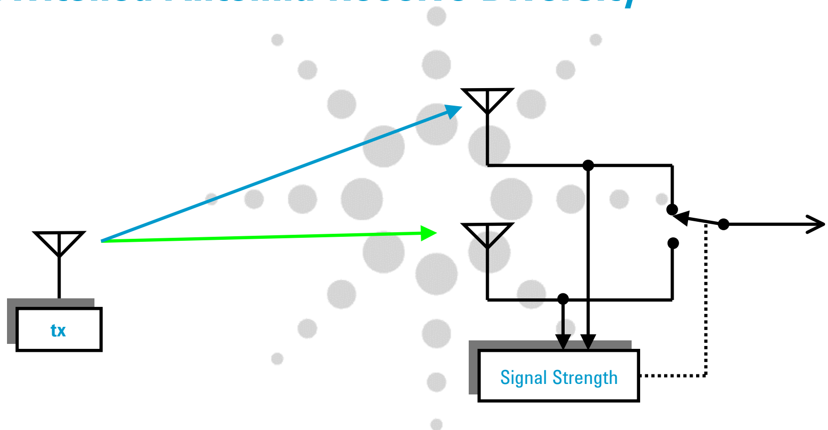
This excellent, and concise definition of Smart Antennas comes from:
http://www.iec.org/tutorials/smart_ant/index.html.

Smart Antenna Technologies

- **Diversity**
- **Beam Forming**
  - Switched Arrays
  - Dynamically Steered Arrays
  - Adaptive Arrays
- **Spatial Division Multiple Access (SDMA)**
- **Multiple Input, Multiple Output (MIMO)**

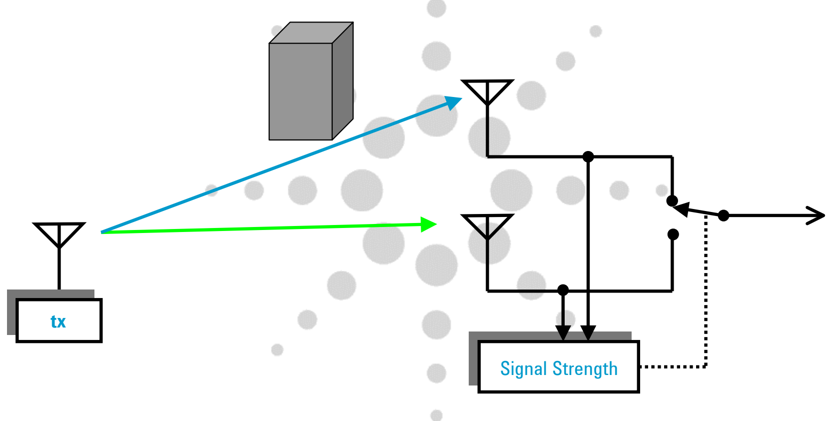
The term “Smart Antenna” is applied to a number of technologies, often creating confusion. Smart antenna technology can be roughly divided into the four different technologies listed here. For each of these technologies, there are variations, such as those shown for “Beam Forming”. Also, more than one of these technologies may be combined within a single system.

Diversity Example
Switched Antenna Receive Diversity



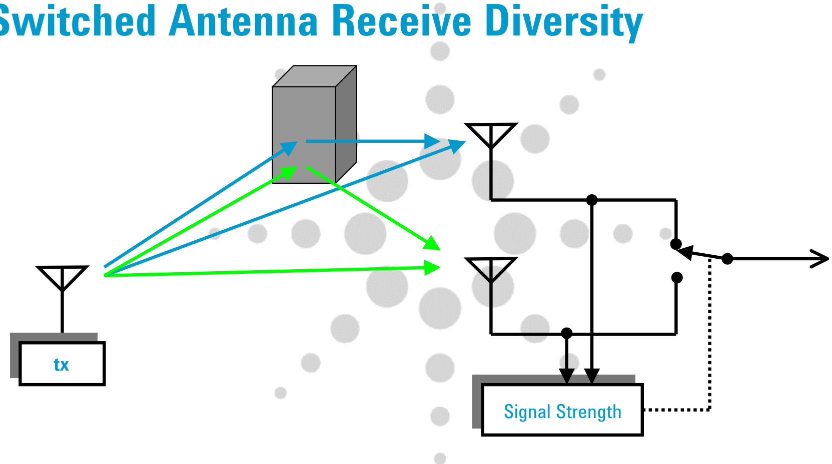
Perhaps the simplest smart antenna is the switched antenna, diversity receive system. Two or more antennas are spatially separated so as to suffer different fading. An algorithm is used to select the antenna with the strongest, or best signal. This technology helps with multipath, but does little to mitigate interference as the antenna is not directional.

Diversity Example
Switched Antenna Receive Diversity



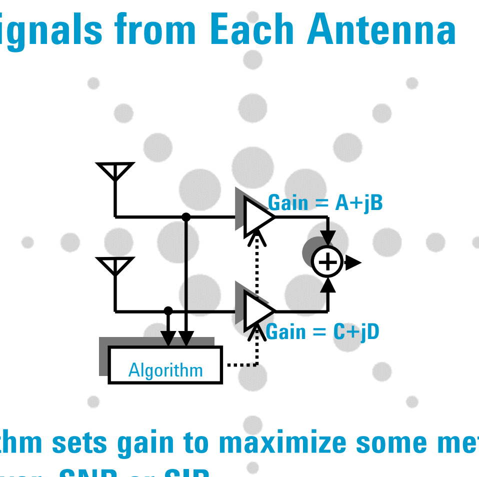
Perhaps the simplest smart antenna is the switched antenna, diversity receive system. Two or more antennas are spatially separated so as to suffer different fading. An algorithm is used to select the antenna with the strongest, or best signal. This technology helps with multipath, but does little to mitigate interference as the antenna is not directional.

Diversity Example
Switched Antenna Receive Diversity



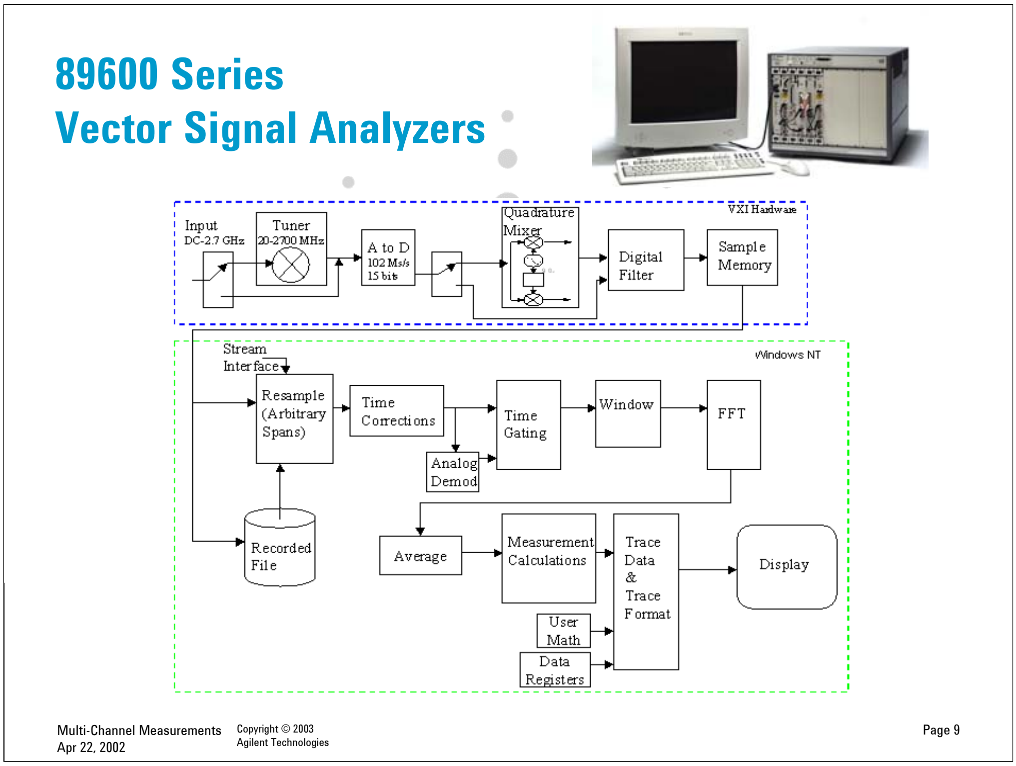
Perhaps the simplest smart antenna is the switched antenna, diversity receive system. Two or more antennas are spatially separated so as to suffer different fading. An algorithm is used to select the antenna with the strongest, or best signal. This technology helps with multipath, but does little to mitigate interference as the antenna is not directional.

Antenna Receive Diversity
Combine Signals from Each Antenna



Algorithm sets gain to maximize some metric such as RX power, SNR or SIR.

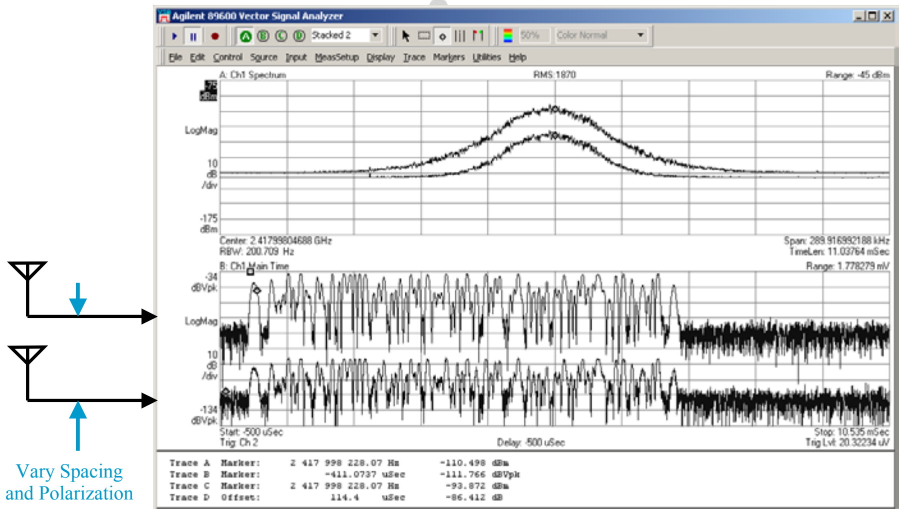
At a cost of increased complexity, performance of the receive diversity system can be improved by combining the signal from two or more antennas. The algorithm used is driven by some metric such as signal-to-noise, or signal-to-interference ratio.



All of the measurements shown in this paper were made using a new 2-RF channel Vector Signal Analyzer, or VSA, very similar to the two-channel baseband analyzer shown in the photograph. The VSA downconverts and digitizes RF signals up to 36 MHz wide. Using a combination of analog and digital filters, the VSA can isolate a signal in a crowded RF spectrum, and store those (complex) samples in memory. Once captured, the signal for each channel can be independently processed, or combined to create new cross-channel results.

Also shown is a block diagram for a single-channel VSA. The two channel model is very similar. What makes the two channel analyzer unique, is that both measurement channels are time aligned and phase coherent. Both of these properties are essential for most of the measurements shown in this presentation. The measurement channels are phase coherent to 2.7 GHz. Above 2.7 GHz the two channels remain frequency locked and time-aligned.

Signal Analyzer Captures From Two Antennas



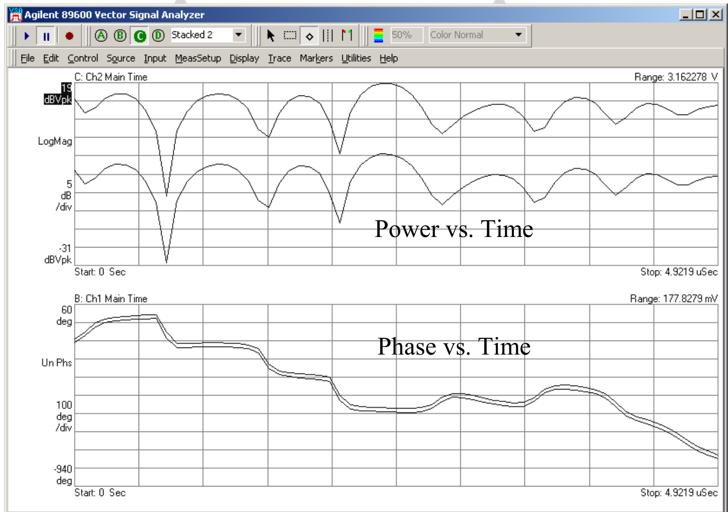
Getting back to the diversity antenna system...

With two measurement channels, and large quantities of capture memory, live signals can be captured for use with simulators, such as Agilent's Advanced Design System (ADS). Antenna spacing and polarization can be varied and the effects on receiver designs observed. This allows smart antenna algorithms to be tested under real-world conditions, as well as the development of better antenna and propagation models.

Here, two antennas were used to capture an off-the-air signal. The lower grid displays the power-verses-time plots for each of the two antennas. As you might expect, they look very similar, especially since this is a very narrow band signal. However from the (averaged) spectrum results shown in the upper grid, you can easily see that the power level at each antenna is quite different.

While multipath itself can be simulated, simulation accuracy can be improved using captured signals, especially when channel and antenna models are not well developed.

Time Waveforms are Complex

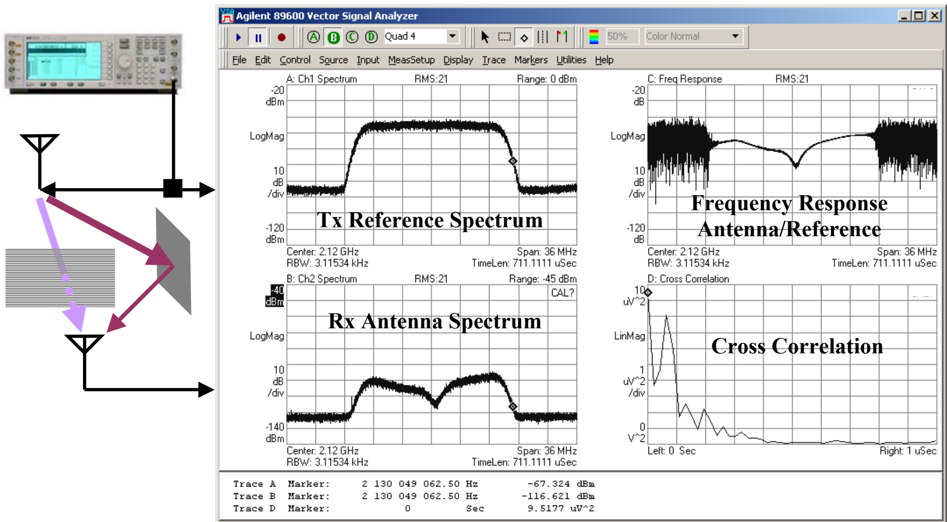


While the time waveforms shown in the previous slide show the signal in magnitude form, the captured signals are complex.

This plot shows two different time waveforms. One measurement channel is connected to the input of an amplifier, the other, to the output. In the upper grid, the two time waveforms are shown in log-magnitude coordinates. In the lower grid, the same two waveforms are displayed in phase coordinates. The display scaling has been adjusted so that the signals are separated vertically, allowing them to be visually resolved.

Clearly, the results from each channel are time-aligned and phase-coherent.

Using Two Channels to Measure Delay Spread



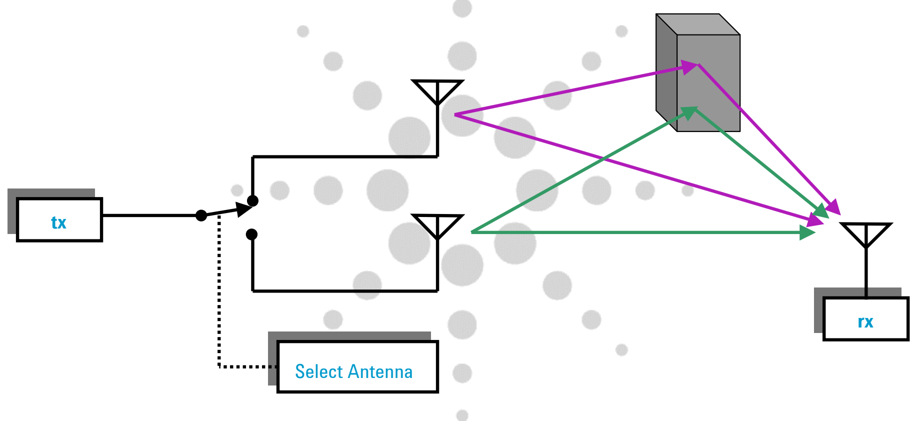
In this measurement, a signal generator was connected through a splitter to an antenna, and to the VSA's channel 1 input. The generator was producing a random noise signal, band limited to 15 MHz. A second antenna was connected to the VSA's second measurement channel. The antennas were separated by ~2 meters with a large metal object partially blocking line-of-site transmission.

The spectrum of the signal coming from the receive antenna clearly shows a spectral notch resulting from multipath in the RF channel. The frequency response result, computed using data from both measurement channels, can be interpreted in this measurement as the channel response. The frequency response is a complex function and is displayed in the upper, right grid in log-magnitude coordinates. Other coordinates, such as phase, or group delay may also be selected.

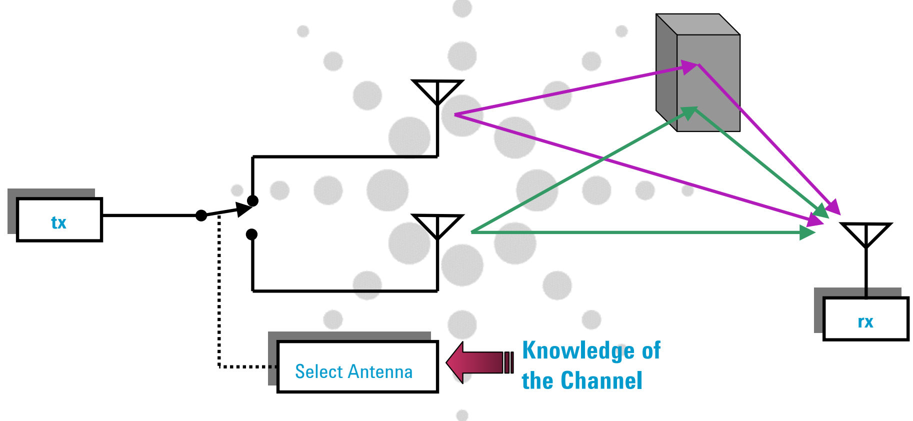
In the lower, right grid the cross correlation between the time waveforms on each measurement channel shows the impulse response and delay spread of the channel. Time resolution in the cross-correlation result is a function of both the signal bandwidth, and the instrument span selected.

Note: If one of the channels can't be connected directly to the transmitter, or placed near the transmitting antenna, then a directional antenna can be used to obtain the reference signal. The directional antenna should be adjusted to obtain the direct-path signal, minimizing the effects of multipath.

Diversity Example
Switched Antenna Transmit Diversity



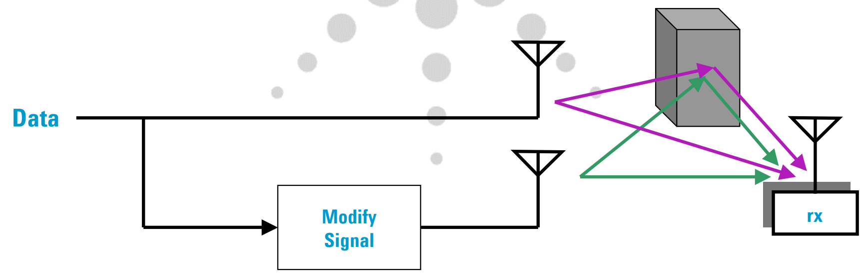
Diversity Example
Switched Antenna Transmit Diversity



Transmit diversity is sometimes used, but not as shown in this diagram. The problem with transmit diversity is that the channel characteristics are unknown to the transmitter unless reported by the receiver. This is usually not done because of the capacity required in the reverse link for updating channel information, and because of the high probability that the channel characteristics will change before the information can be used.

Transmit Diversity

- **Usually Done without knowledge of the channel**
- **Same information is transmitted on each antenna, but signals on each antenna are different in some respect. (If they were the same, we'd call it beam forming!)**

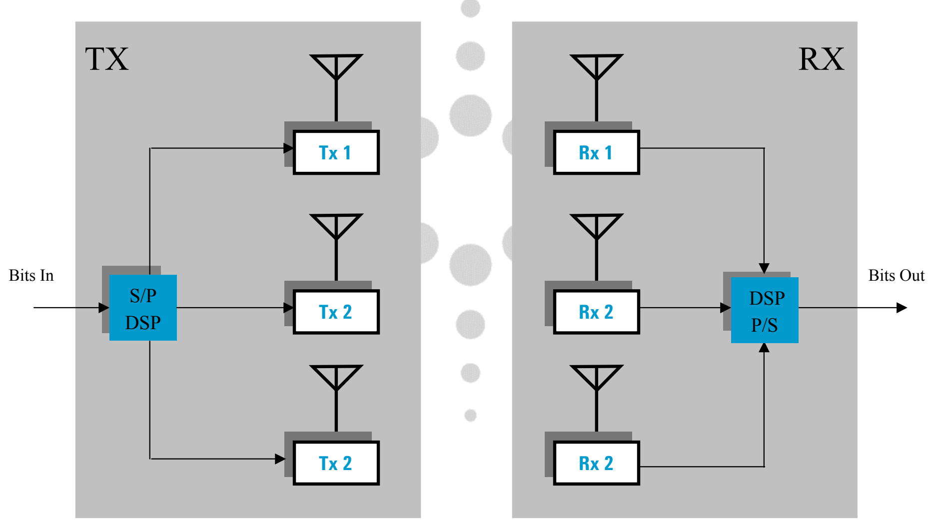


Transmit diversity is usually accomplished using an approach where the exact same information is transmitted on two antennas, but with some important difference. For example, the signal on the second antenna could simply be the signal on the first antenna, delayed a few symbols in time. Space-time coding is often used to in transmit diversity (and MIMO) systems.

At the receive antenna, the received signal will be some linear combination of the two transmitted signals. Provided that Tx antenna spacing is sufficiently large, the signal from each transmit antenna will suffer from different multipath by the time it reaches the receiver. The probability that both signals will, simultaneously, be completely faded at the receiver is reasonably low. To make sense of the combined signals, the receiver uses information unique to each signal to estimate the channel from each Tx antenna to the Rx antenna.

For this type of system, a two channel VSA may be used to verify that the modified signal path has the correct relationship to the direct signal path. If a repetitive test signal is used, and a trigger is available, a single channel instrument can be used. In general, precise time alignment is not required for data transfer, though it might be required for other reasons such as E911 geolocation.

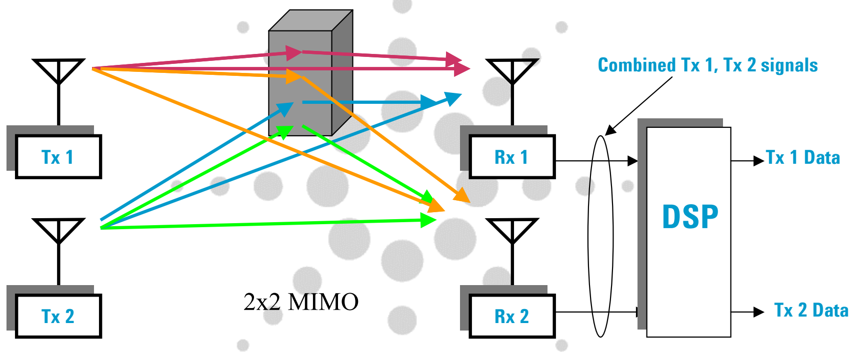
MIMO: Want to increase data capacity by 3X? Triple the number of TX/RX Pairs!



3x3 MIMO

A fourth type of smart antenna technology is referred to as MIMO, for Multiple-Input, Multiple Output. It gets its name from the use of multiple antennas at the transmitter (input to the channel) and multiple antennas at the receiver (output of the channel). Unlike all of the other smart-antenna systems, in this system each antenna is transmitting a unique data stream. Here, the primary goal is to increase the data rate, as opposed to increasing the number of users through beam forming.

MIMO: Theoretical capacity increases linearly with the number of Tx/Rx antennas



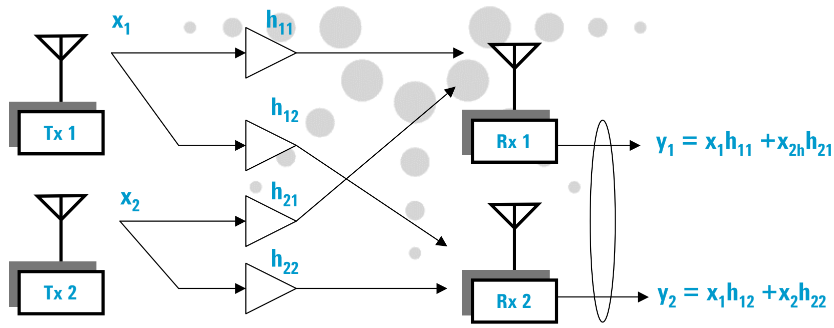
Unlike SDMA, each transmitted signal is completely unique. Each received signal represents the linear summation of the transmitted signals. With knowledge of the channel, signals are separable.

Most of these systems rely on multipath to ensure that each receiver sees a different linear combination of transmit signals. When that condition is met, the receiver can estimate the channel characteristics from each and every transmit antenna, to each every receive antenna and then use that information to isolate each transmit signal. For example, in a system with three transmit antennas and three receive antennas, nine channel responses would need to be estimated. In theory, this system can increase capacity by the number of transmit/receive pairs. So a four antenna system could have 4 times the capacity of a system which only used one antenna at the transmitter and one at the receiver.

Since beam forming isn't the goal of this system, precise amplitude and phase match between the antennas is much less critical. However, better performance can be achieved with phase-coherent channels. A two channel analyzer can be used to capture information from two closely spaced receive antennas, or between Tx/Rx pairings, to gather data for building MIMO propagation models.

How It Works

In this example, we ignore multipath induced ISI. This is fair if we select a OFDM as our modulation format. For OFDM subcarrier, each path can be modeled as a simple gain scaling and phase rotation, i.e. a single complex multiply.



Simple Linear Algebra...
... If you know "H"

$y_1 = x_1h_{11} + x_2h_{21}$

$y_2 = x_1H_{12} + x_2H_{22}$

In Matrix Form

$Y=HX$

Solving for X

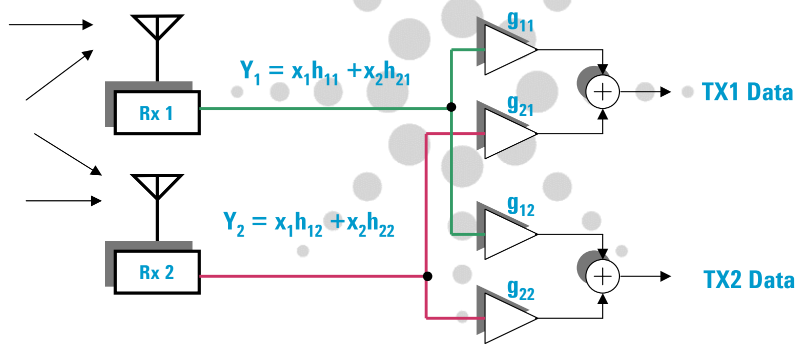
$X = H^{-1}Y$

With the assumptions made, we can treat this as a two-equation, two-unknown problem. Inverting the matrix 'H' requires that the channels be independent. This will occur if the antennas are spaced far enough apart and there is sufficient multipath.

Note that we've assume 'H' is known. It usually isn't and must be estimated from known information imbedded in each signal.

How It Works (cont)

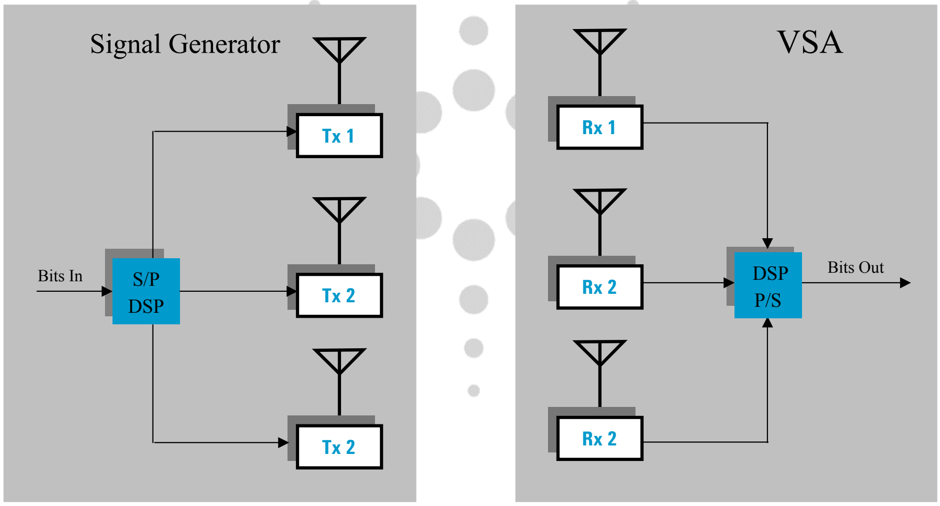
Let $G=H^{-1}$



Next Step: Demodulate each channel and recombine data streams

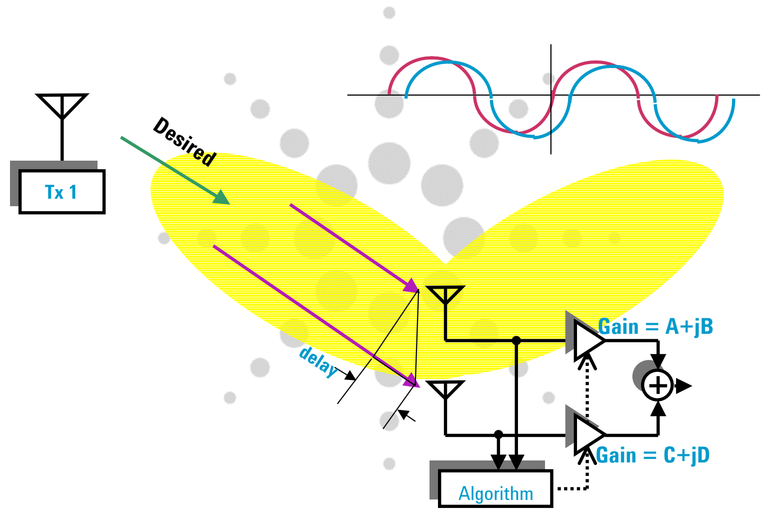
Once we have inverted H to obtain G, we can recover the original signals.

For MIMO, *we* need as many channels as the customer if we want to work at the bit level

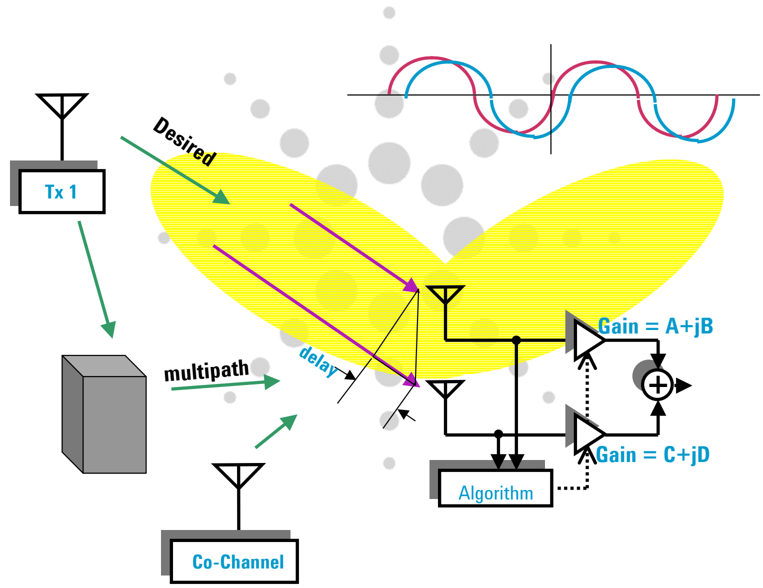


While it may be possible to analyze each transmit signal independent of the others for EVM measurements and the like, it will be necessary to simultaneously demodulate all of the channels to obtain the composite bit stream.

Beam Forming



Beam Forming



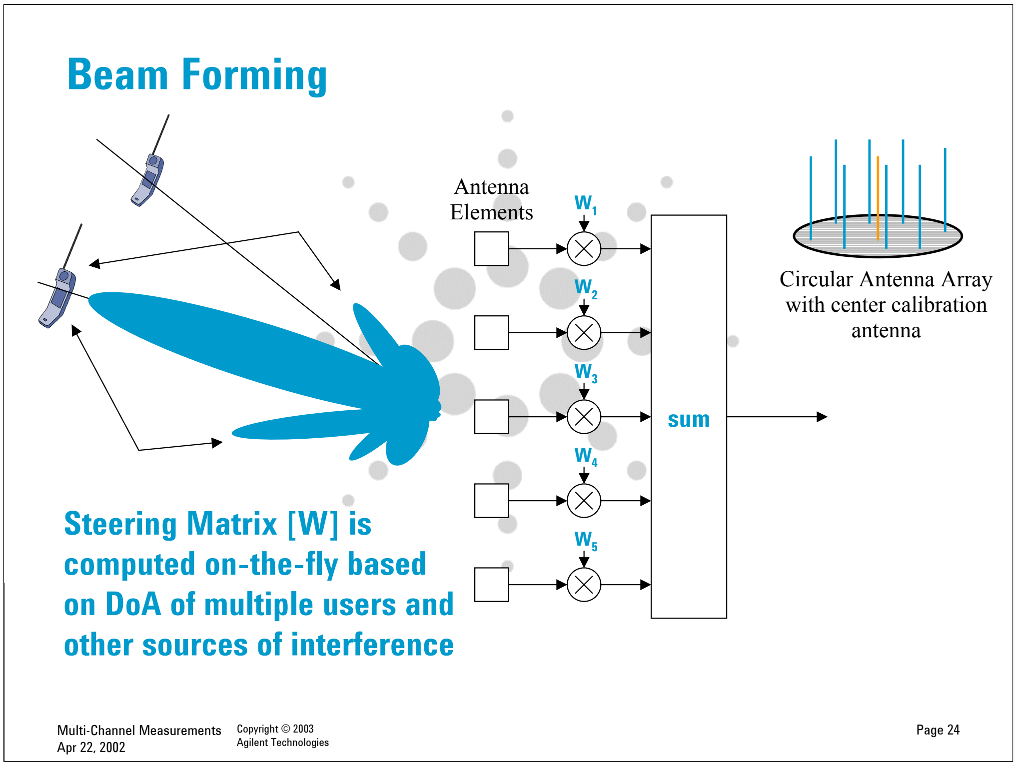
Beam forming can be used on both transmit and received signals. For example a base station (BTS) may form a beam when transmitting to a mobile. Since the entire sector doesn't need to be illuminated, less power is required, improving overall system performance. The base station receiver will also use beam forming to increase antenna gain in the direction of the mobile (this also provides the BTS with information on which direction to transmit). On the receive side, off-axis co-channel interference and multipath is attenuated, again improving system performance.

Beam forming algorithms can often be very simple, For example the gain coefficients can be pre-computed to provide a fixed set of antenna patterns. The algorithm then simply switches between patterns until the best signal is found.

Beam forming and switched beam systems are compatible with frequency division duplex (FDD) systems.

Note 1: Most "switched-beam" systems don't use electronic (DSP) steering, but use some form of switch-signal phasing, such as a Butler Matrix. Those systems are less likely to require multi-channel signal analysis.

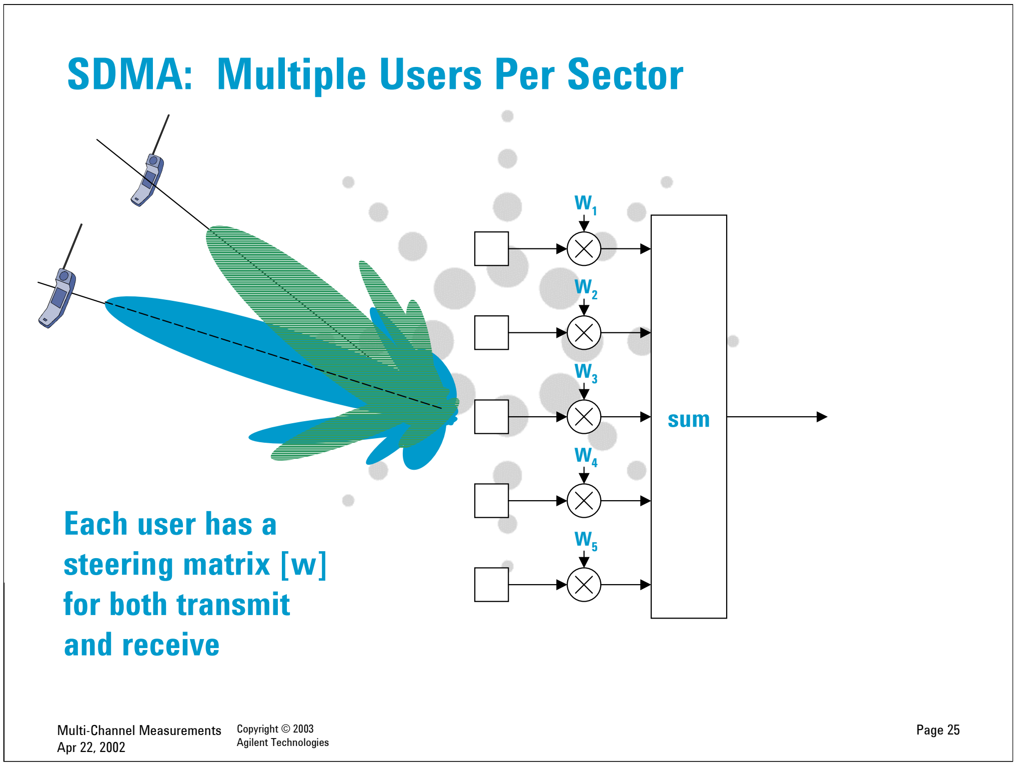
Note 2: The delay in this diagram is very short, usually less than one cycle of the carrier. In measurements, this delay presents itself as a carrier phase rotation. Much larger delays, on the order of 1/(Signal Bandwidth), are observable as a change in phase as a function of frequency (phase and group delay measurements)



A more advanced form of beam forming will attempt to steer the main lobe of the antenna pattern, as well as the sidelobes. This technique takes advantage of the multipath to maximize energy transfer. A rake receiver or equalizer might then be used to compensate for the temporal distribution in the signal.

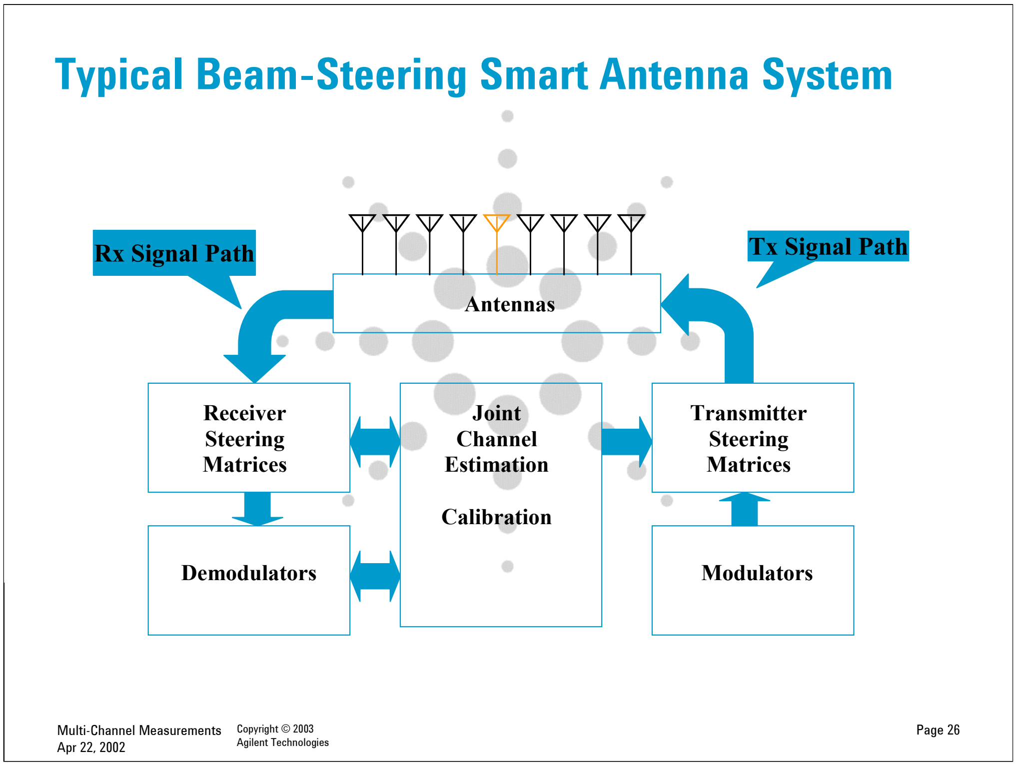
A further refinement on the beam forming approach is to estimate the direction of arrival (DoA) of co-channel interference, and then steer nulls to attenuate the interfering signal.

These more advanced forms of beam forming work best with Time-Division-Duplex (TDD) systems. Most communications systems are bi-directional. If both radios are on the same frequency, then channel reciprocity can be assumed. That is to say that the multipath looks the same in both directions. Since multipath is a strong function of frequency, channel reciprocity cannot be assumed in FDD systems.



Spatial Division Multiple Access (SDMA) attempts to reuse the same channel two or three times within a sector. Here a channel is not just a frequency channel, but could also be the same code channel, or time slot.

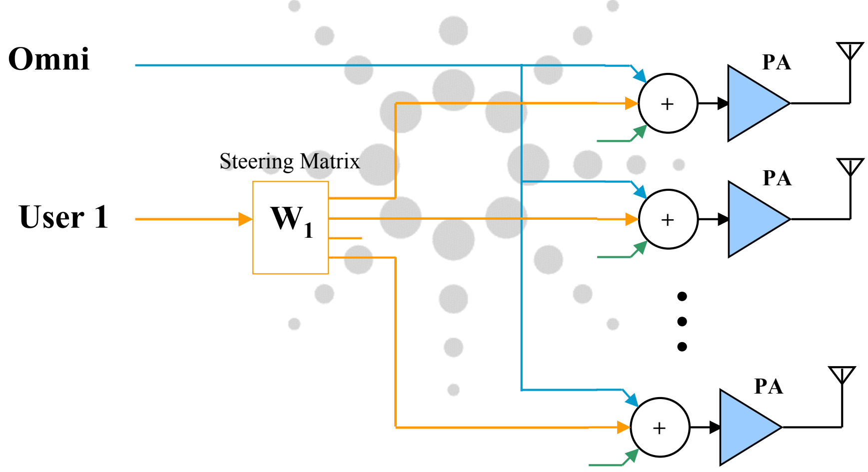
While this slide shows formed beams for each user, the transmit pattern may not be so easily described. The primary goal of SDMA is for each (spatial) channel to have constructive interference at the location of the intended receiver and destructive interference at the location of all other receivers.



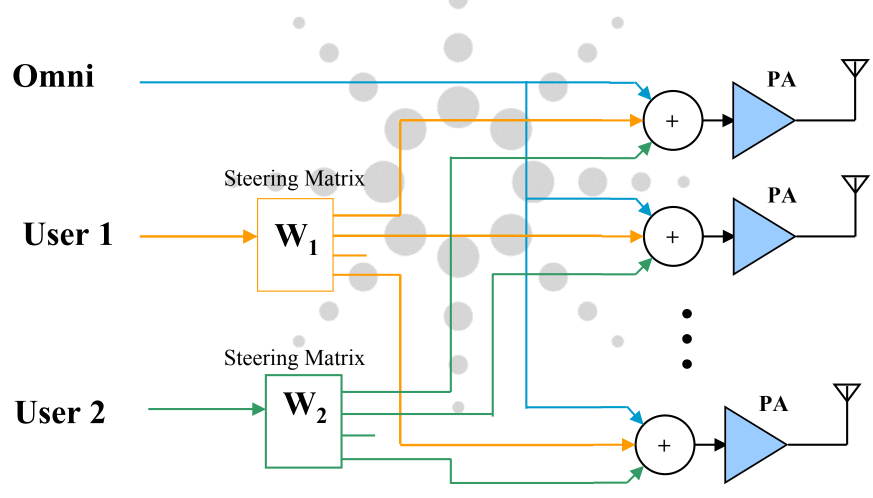
Let’s now take a look at beam-steering systems to see how multi-channel RF measurements can be used to characterize and troubleshoot smart-antenna systems.

For TDD/SDMA systems channel reciprocity can be assumed, at least for the part of the channel between transmit antenna and receive antenna. Within the radio itself, reciprocity is not a safe assumption. As you can see in this diagram, the potential exists for the signal path in the receiver to be different from the signal path in the transmitter. For SDMA systems, imperfect match will result in interference between users. To help with this problem, a calibration antenna may be built into the smart-antenna system.

Amplifier gain/delay must remain stable between calibrations for accurate beam forming



Amplifier gain/delay must remain stable between calibrations for accurate beam forming



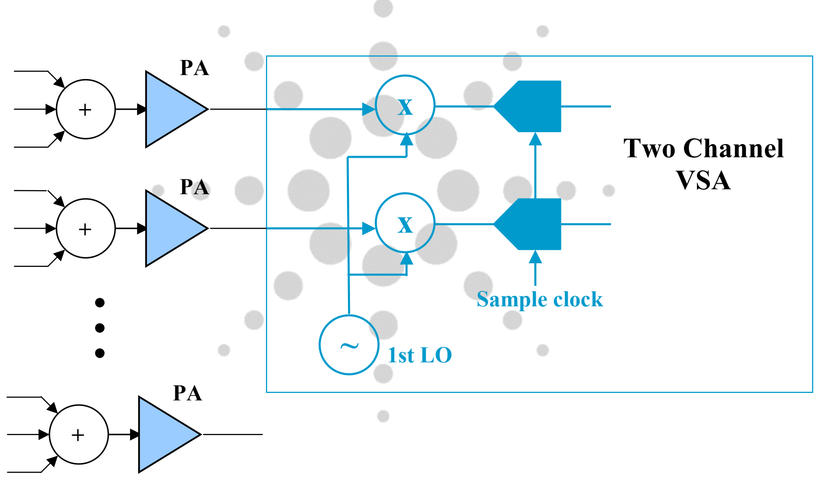
Multi-channel measurements can be used to verify the performance of smart-antenna calibrations. They can also be used to measure the performance of parameters that may not be calibrated. Here's an example:

For cost reasons, users must share amplifiers and antennas. In the system shown above, there are three signals: a beacon signal, which is transmitted in all directions (so it's steering matrix may have all coefficients set to one), and two users. When the second user became active, the average power level in each PA increased, however the increase usually isn't the same for each amplifier, as the steering matrix usually won't have the same amplitude for each antenna.

It's important that as the average signal power changes, the gain and phase performance of the amplifiers remain matched. Mismatches will affect the performance of the system, causing interference between users as the radiation patterns degrade.

Coherent Multi-Channel Measurements:

Common LO's (analog and digital), ADC sample Clock



Using a two channel VSA, it is possible to compare the signals coming from each amplifier. Notice that the VSA uses a common 1st LO and a common ADC sample clock. This is very important for phase coherent measurements. *This measurement could not be performed if each receiver had it's own LO, even if sharing a common frequency reference.*

While traditional network analyzers can, and have been used to make this measurement, the requirement to use sinusoidal signals can be very limiting. The multi-channel VSA approach allows any signal to be used, with any power statistics. In addition, the signal does not need to be repetitive. This allows the system to be tested under very close to normal operating conditions.

Possible Amplifier Measurements

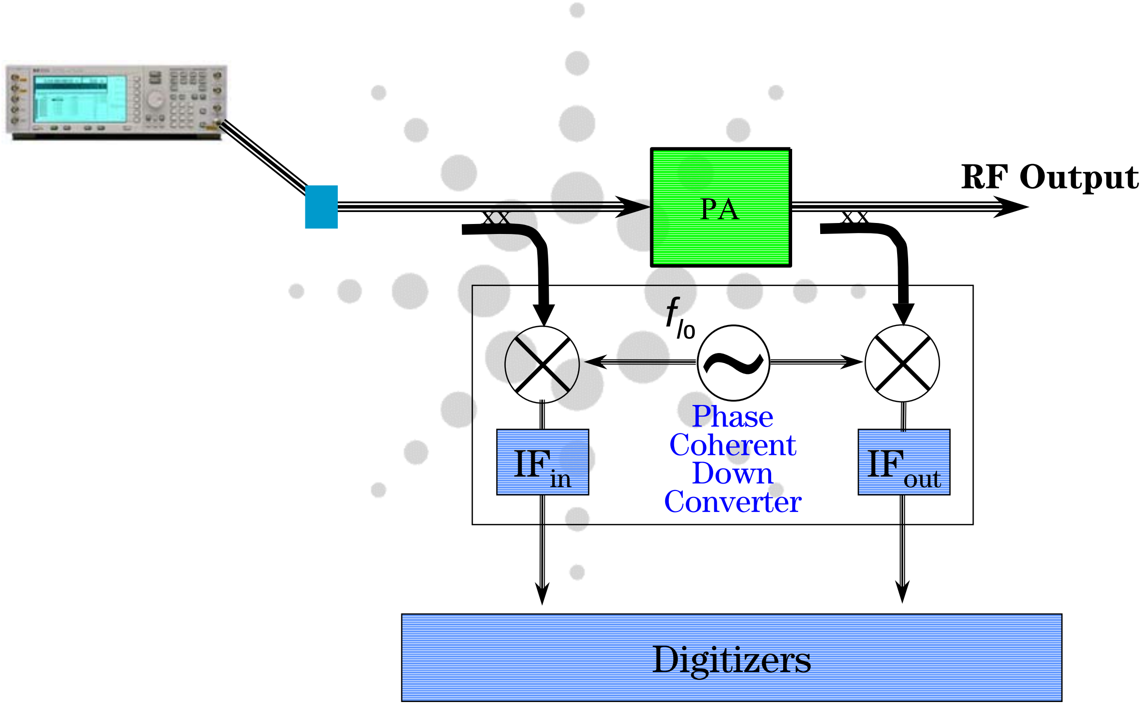
- Absolute Mag/Phase relationships between channels
- Relative Mag/Phase relationship between channels

  Over time, to observe amplitude and delay instability in the amplifiers (due to thermal drift, active compensation, etc.)

  As a function of signal power. Use cross-channel measurements (e.g. frequency response) to detect delay changes caused by variations in the output power of one amp relative to another.

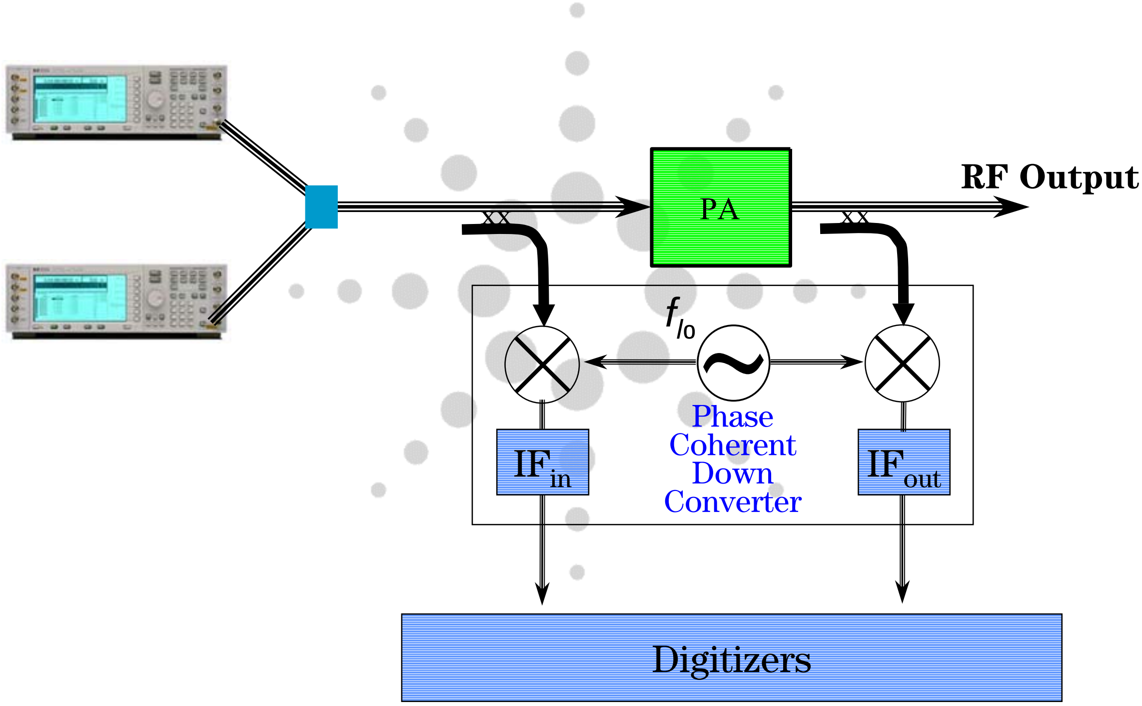
- All of the above, except between the input and output of an amplifier or other device

Measurements on Nonlinear Devices



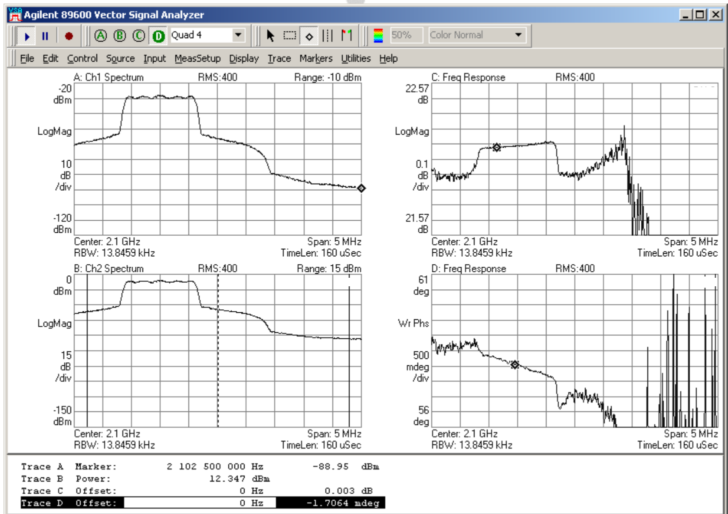
A two channel measurement can be made between the outputs of two different amplifiers, or between the input and output of an amplifier as shown here. In this example, two signal generators are used to create two CDMA2000 signals which are combined and sent to the PA. Two signal generators were used as it was more convenient to quickly turn the CDMA signals on and off, and to adjust power levels.

Measurements on Nonlinear Devices



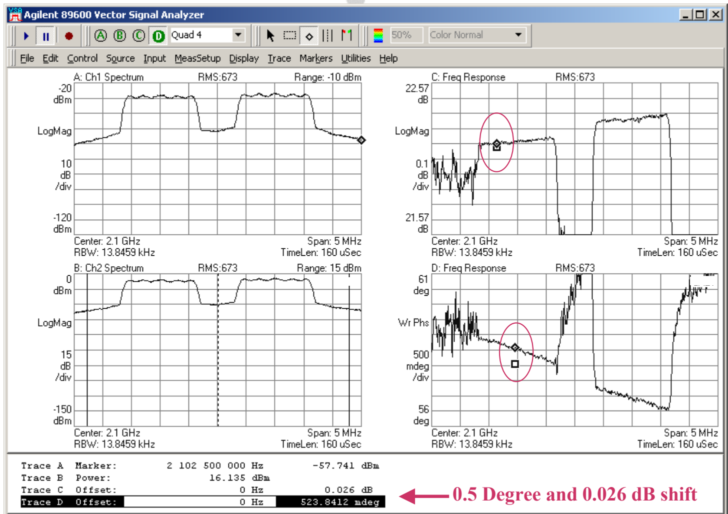
A two channel measurement can be made between the outputs of two different amplifiers, or between the input and output of an amplifier as shown here. In this example, two signal generators are used to create two CDMA2000 signals which are combined and sent to the PA. Two signal generators were used as it was more convenient to quickly turn the CDMA signals on and off, and to adjust power levels.

Baseline Frequency Response Measurement



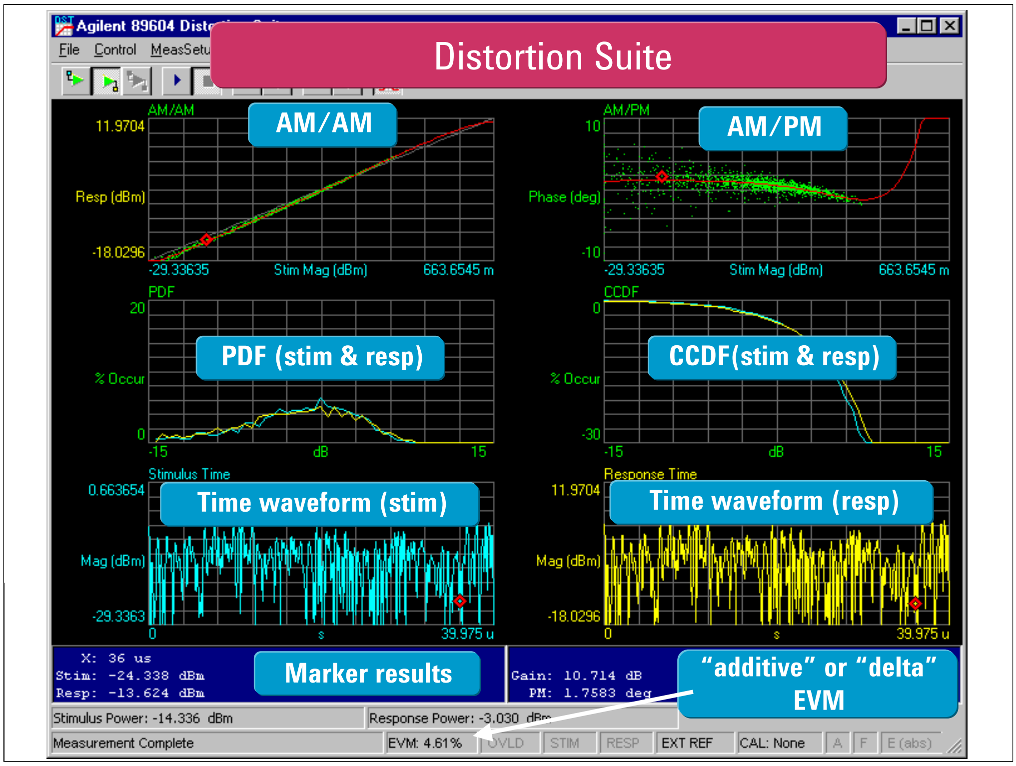
With one signal generator enabled, a frequency response measurement was made. The traces on the right show the magnitude and phase response. For the purposes of this measurement, the shape of the frequency response function is not important. What is important is how this result will change when a second RF signal is added, increasing the load on the amplifier.

Second CDMA Signal Added



Here the second CDMA200 signal has been added, increasing the output power by 3.8dB overall. From the offset markers you can observe that the phase of the original signal has shifted upwards by by 0.5 degree, and the gain has increased by 0.026dB. To put the phase shift in perspective, 0.5 degrees at 2.1GHz represents a delay shift of 0.66 psec.

Is this significant? Probably not if the smart antenna application is trying to steer a main lobe. It may be significant if the goal is to steer energy away from a co-channel user. This is also one element of a total error budget that includes other active and passive components and cabling.



Agilent's Distortion Suite software can be used with two-channel systems. This software package provides many results useful to designers of power amplifiers.

While the software package also supports single-channel analyzers, two-channel analyzers may offer a few advantages:

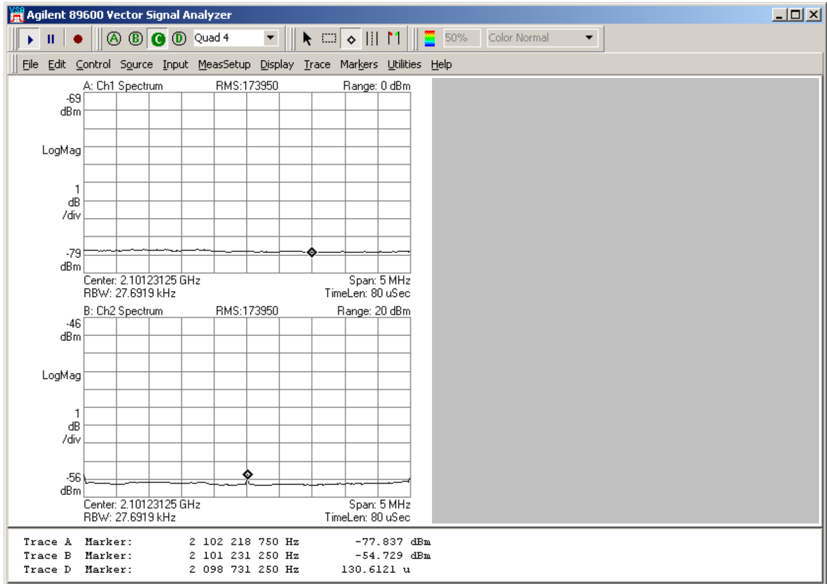
1. They don't rely on triggering and repetitive stimulus signals

2. With two single-channel measurements, there's always the possibility the the DUT will change in the interval of time between measuring the input signal and the output signal.

3. If the two measurement channels are phase coherent, close-in phase noise is mostly cancelled between the measurement receivers.

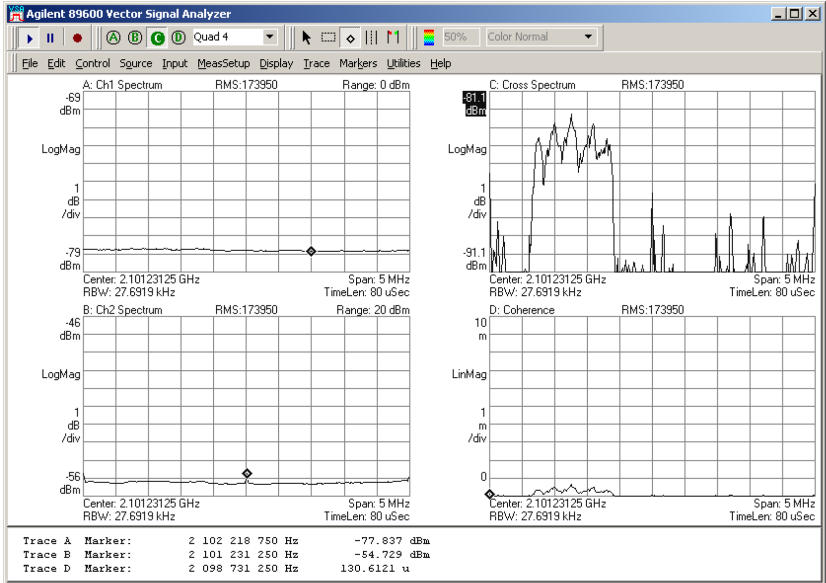
4. Phase noise in the source is simply part of the signal seen by both receivers.

Finding Hidden Signals



Shown hare are the spectrums for two measurement channels. They both contain a very small amount of the same signal. Can you tell where the signal is? Can you tell what it is? Note that the vertical scaling is 1dB/div!

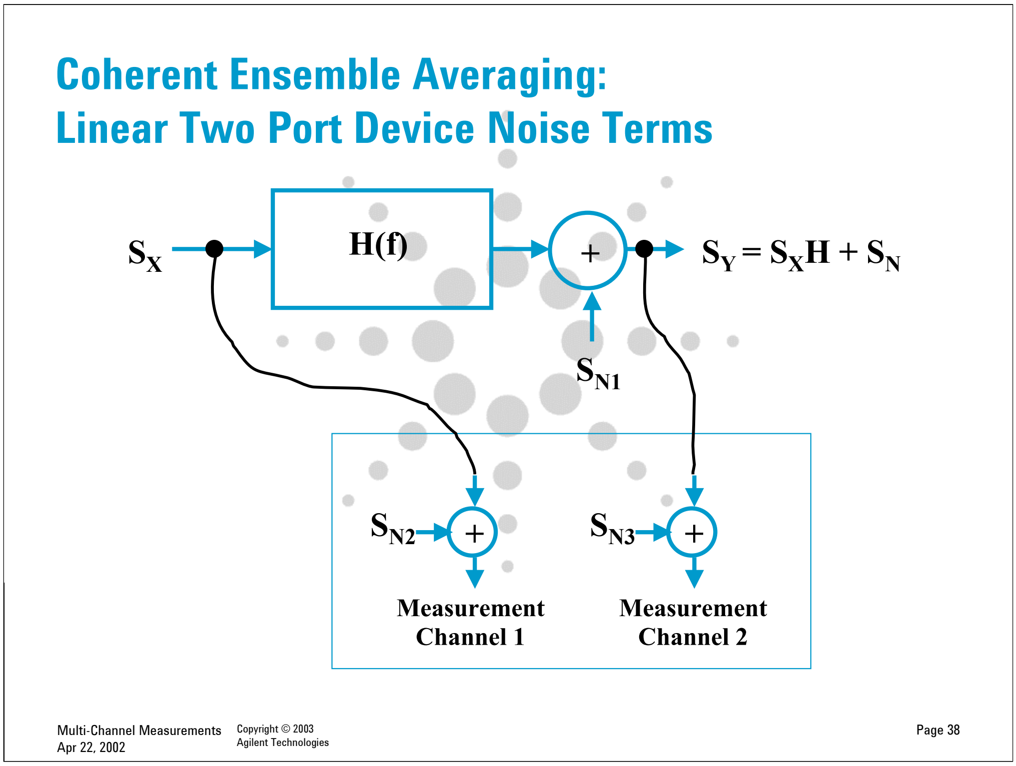
Finding Hidden Signals



The CDMA2000 signal is very easy to spot in the cross-spectrum measurement shown in the upper, right grid. You can even observe the passband ripple of the transmit filter.

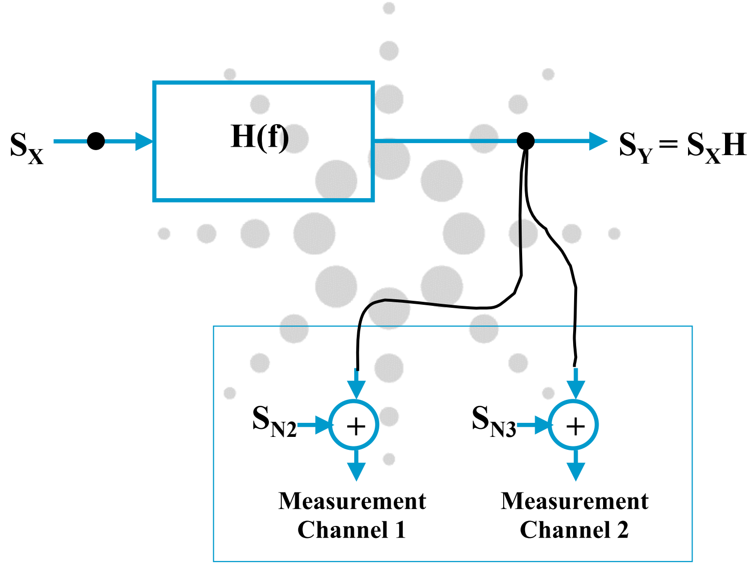
This slide shows the power of having more than one measurement point. By observing a signal at two different points in a circuit, or in space (using two antennas), correlated elements of two signals can be reinforced, while uncorrelated elements (not just noise) can be suppressed.

This technique can be used, as shown here, to increase dynamic range, to find the origin of unwanted signals, or to discover unwanted signals in the presence of larger desirable signals.



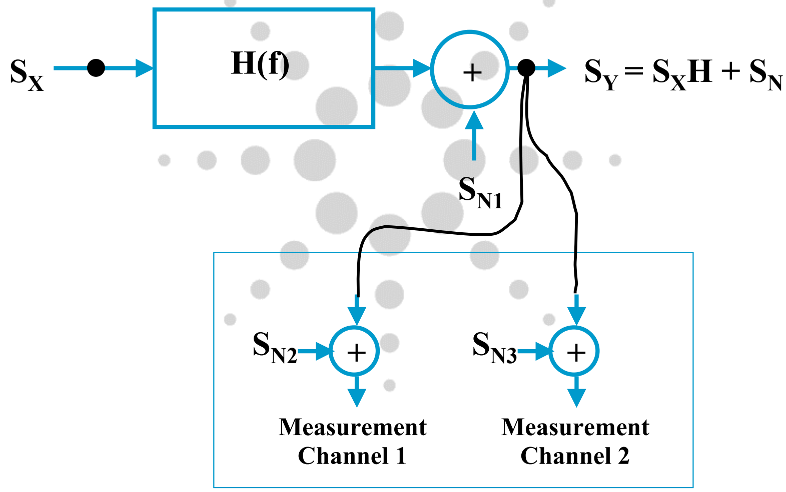
This is the setup used to make the previous measurement. In that measurement, H(f) was a power-amplifier. Notice there are three sources of noise. The amplifier is modeled as having a noise source, as is each measurement channel. The cross-spectrum measurement shown in the previous slide attenuates any signal that is not common to both channels. In this case, each noise source is only seen by one channel, so all of the amplifier and measurement noise is attenuated (not just smoothed) with averaging.

Overcoming Instrument Noise

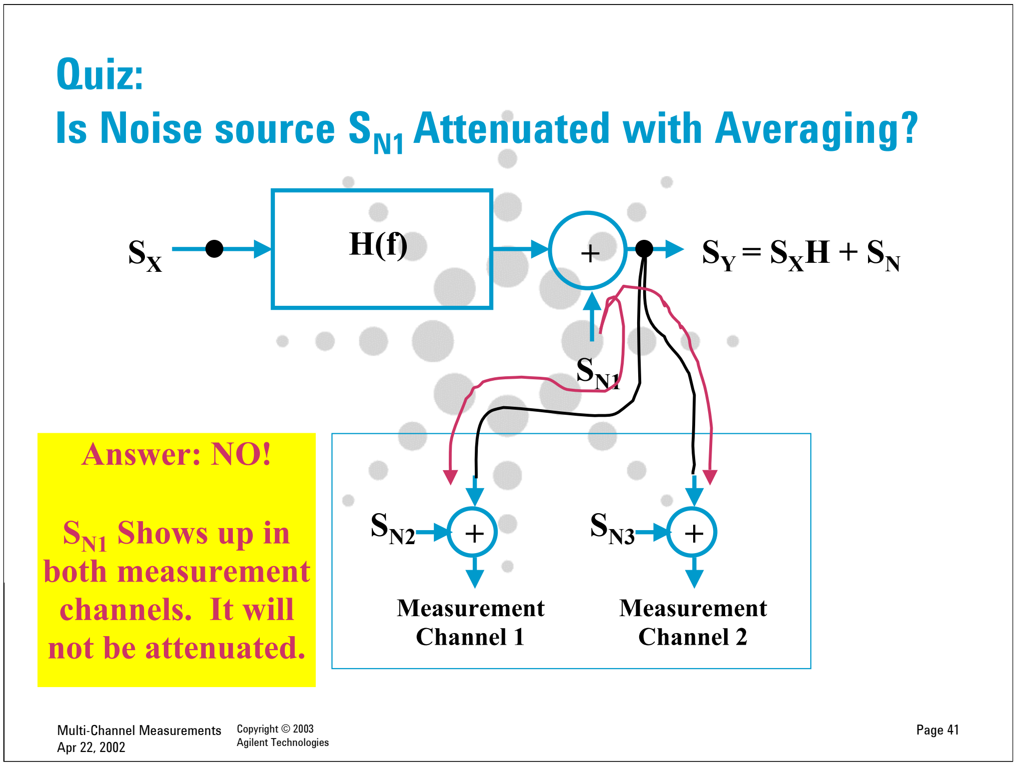


This technique can be used to enhance the dynamic range of a measurement that is noise limited (as opposed to distortion limited).

Quiz:
Is Noise source S_{N1} Attenuated with Averaging?

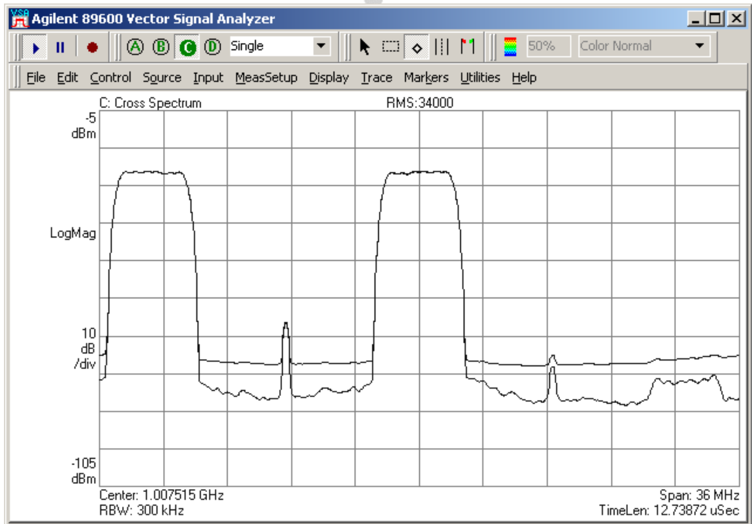


This is the setup used to make the previous measurement. In that measurement, H(f) was a power-amplifier. Notice there are three sources of noise. The amplifier is modeled as having a noise source, as is each measurement channel. The cross-spectrum measurement shown in the previous slide attenuates any signal that is not common to both channels. In this case, each noise source is only seen by one channel, so all of the amplifier and measurement noise is attenuated (not just smoothed) with averaging.



This is the setup used to make the previous measurement. In that measurement, H(f) was a power-amplifier. Notice there are three sources of noise. The amplifier is modeled as having a noise source, as is each measurement channel. The cross-spectrum measurement shown in the previous slide attenuates any signal that is not common to both channels. In this case, each noise source is only seen by one channel, so all of the amplifier and measurement noise is attenuated (not just smoothed) with averaging.

Overcoming Instrument Noise (cont)



Since both measurement channels are looking at the same signal, the noise on each channel can be removed from the measurement. Here, it's much easier to see the third-order distortion product of this multi-carrier signal in the cross-spectrum measurement (lower trace) than in the single-channel spectrum measurement (upper trace).

What is Noise?

White Noise:

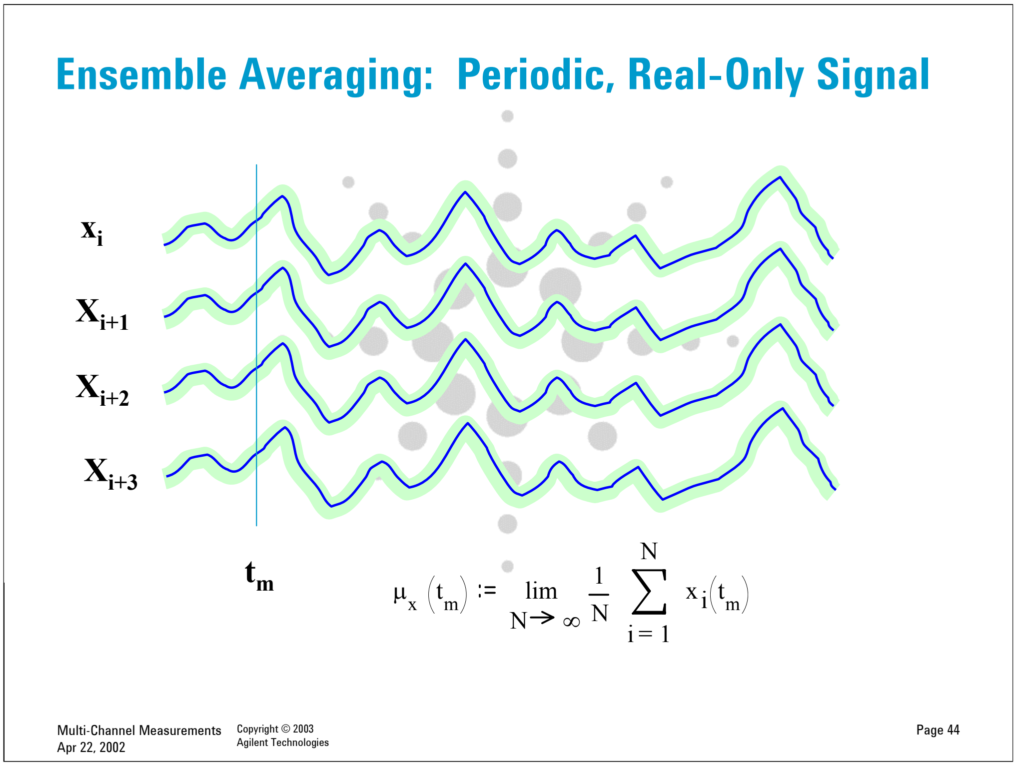
Autocorrelation = 0 except for zero offset. No assumption about voltage distribution

Gaussian Noise:

Usually assumed to be white (AWGN), voltage statistics are Gaussian

Bandlimited Gaussian Noise:

Correlated over time offset determined by bandlimiting function. Bandlimiting allows complex signal representation with Gaussian voltage statistics for I and Q. Power statistics are chi-square with 2 degrees of freedom



Ensemble averaging is a technique used to mitigate uncertainty do to noise. Here, four different measurements of a time waveform are shown. Obviously the time waveform is repetitive over some interval. To get a better estimate of the voltage of the waveform at a particular time, the four waveforms are averaged together.

While this slide demonstrates how ensemble averaging can be used to mitigate noise on time waveforms, the technique can be applied to any ensemble of measurements where, except for noise, the results should be the same.

With single-channel measurements, ensemble averaging can be used to reduce noise on triggered, phase-coherent measurements. When triggering isn't used, and the measurements aren't coherent, averaging can still be used to reduce the variance due to noise, but not to reduce the noise itself.

With dual-channel time-aligned and coherent measurements, many of the results can be averaged to reduce noise, even without triggering. The cross-spectrum and frequency response measurements are two good examples.

Ensemble Averaging Errors (uniform PSD)

Random Error when estimating the mean $\frac{\sigma_x}{\mu_x \cdot \sqrt{N}}$

Random Error when estimating the variance (power) $\sqrt{\frac{2}{N}}$

N	-10log(N)
10	-10 dBr
20	-13 dBr
50	-17 dBr
100	-20 dBr
1000	-30 dBr

Practically, the effects of noise can never be completely eliminated. Every 10 dB improvement requires 10 times the number of averages. Fortunately with today's fast computers, measurements with one hundred thousand averages are possible in a reasonable amount of time.

Basic Statistics For Ensemble Averaging

Mean

$$\mu_x(t_m) := \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} x_i(t_m)$$

$$\mu_x(f_m) := \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} x_i(f_m)$$

Mean Square

$$\psi_x^2(t_m) := \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \left(\left| x_i(t_m) \right| \right)^2$$

The first equation is the average of a time waveform. The second equation might represent, for example, frequency bins in a cross-spectrum measurement.

For VSA's, the quantity being averaged is usually complex.

Also note the location of the indexing variable i. In words, N is the number of measurements.

Basic Statistics For Time Series Data

Mean
$$\mu_x := \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} x(t_i)$$

Mean Square (power)
$$\psi_x^2 := \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \left(\left| x(t_i) \right| \right)^2$$

Variance
$$\sigma_x^2 := \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \left(\left| x(t_i) - \mu_x \right| \right)^2$$

These equations are more familiar to most people and are included here for completeness. While they look similar to the ensemble averaging equations, there is an important difference. The equation for ensemble mean (previous slide) is used to reduce the effects of noise at a specific point in time or frequency. The mean of a time series is an estimate of DC. In these equations, N might represent the number of samples in a time record.

The mean-square over time is often a more interesting value to compute as it is a measure of power in a signal. As we will see on the next slide, the mean-square power is a degenerate form of the auto- and cross-correlation functions (time offset = 0). Understanding this relationship may help in the interpretation of correlation measurements.

Basic Statistics For Time Series Data

Auto Correlation

$$R_{xx}(\tau) := \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} x(t_i) \cdot \overline{x(t_i + \tau)}$$

Cross Correlation

$$R_{xy}(\tau) := \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} x(t_i) \cdot \overline{y(t_i + \tau)}$$

Covariance

$$C_{xy}(\tau) := \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \left(x(t_i) - \mu_x\right) \cdot \overline{\left(y(t_i + \tau) - \mu_y\right)}$$

$$\sigma_{xy} = C_{xy}(0)$$

Auto- and cross-correlation functions are very important to digital communications. Earlier we used the cross-correlation function in a two channel measurement to look at strong reflections and delay spread in an RF channel. In that measurement x(t) represents the time data from one channel, and y(t), the other channel. The cross-correlation measurement was used to find time-shifted copies of the reference signal in the received signal.

Multi-Channel Measurements: Network Analysis

•Traditional

Sinusoidal stimulation

Noise reduction via integration

•Using Complex Signals

Noise reduction: Possible to average cross-channel products.

Any signal can be used to stimulate the DUT.

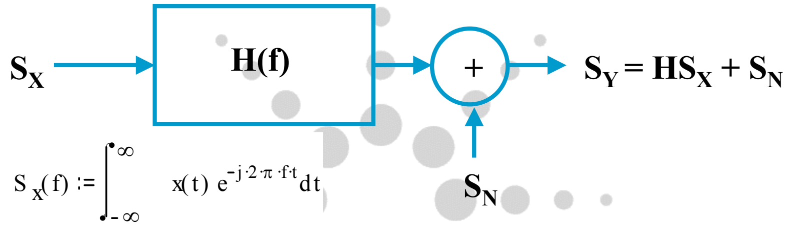
Using noise-like stimulus signals, a good linear model is possible

Some DUT's don't react well to sinusoidal stimulation

Measurements possible in the presence of other, uncorrelated signals

With two coherent channels, it's now possible to do network analysis using completely arbitrary signals.

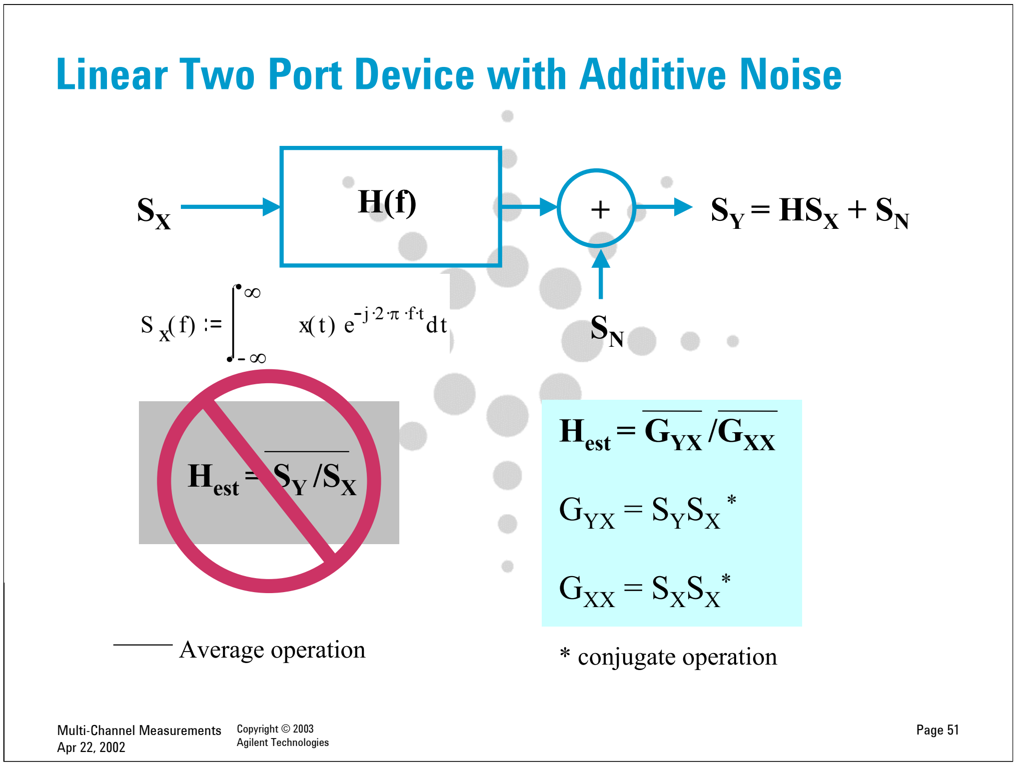
Linear Two Port Device with Additive Noise



$$S_X(f) := \int_{-\infty}^{\infty} x(t)\, e^{-j \cdot 2 \cdot \pi \cdot f \cdot t}\, dt$$

$$H_{est} = \overline{S_Y / S_X}$$

—— Average operation



Measuring the frequency response of a device using broadband stimulus signals and broadband receivers requires careful attention to noise sources and signal characteristics.

For example, there's more than one way to estimate H(f) from the input and output measurements. Here we show that a proper way to estimate H(f) is by averaging the cross-spectrum (G_{XY}) and input auto-spectrum (G_{xx}) separately and then computing the ratio of those two results.

Why not just compute AVG(S_Y/S_X)? For most signals, the spectrum S_X will not have a uniform spectrum over the finite observation interval of a single measurement. If, for a given measurement, the denominator $S(f_1)_X$ is at or near zero, then the estimate of $H(f_1)$, for that measurement will be very unreliable. With the approach shown, the denominator is averaged before it is divided into the numerator.

Use Auto- and Cross-Spectrum averaging to reduce the effects of noise

$$\mathbf{E[G_{XX}] = E[S_X S_X^*]}$$

E[] is the expected value

$$E[G_{YX}] = E[\, S_Y S_X^{\,*} \,]$$

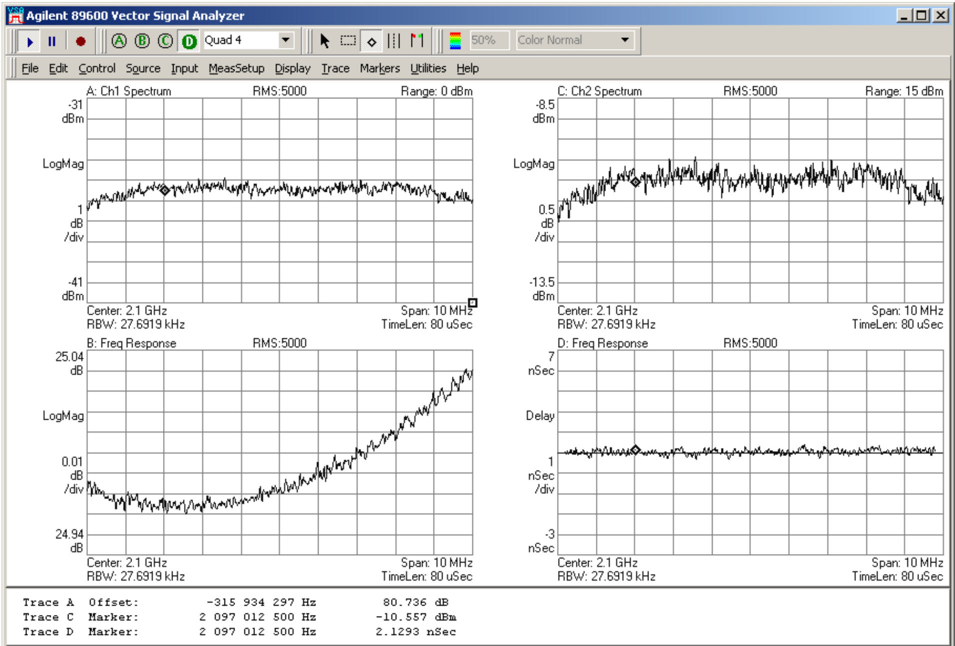
$$= E\,[\, (HS_X + S_N)S_X^{\,*} \,]$$

$$= E\,[\, H\, S_X S_X^{\,*} \,] + E\,[\, S_N S_X^{\,*} \,]$$

0 — Assumes the noise is uncorrelated to the signal

When the expected value of the cross-spectrum between the noise and input signal is zero, $H_{est}(f)$ can be accurately determined in the presence of noise, with sufficient averaging.

The measurement noise shows up in the numerator, but doesn't bias the answer, and can be removed with averaging. If the equations had included the term $S_N S_N^{\,*}$, then the result would have been biased by the noise power spectrum.



Multi-Channel Measurements
Apr 22, 2002

Here's a measurement of H(f), a 2.1 GHz power amplifier. Broadband random noise was used as a stimulus signal. Shown from left to right, and top to bottom are the input spectrum, output spectrum, frequency response magnitude, and frequency response group delay.

This measurement was first calibrated using a "through connection" to minimize error due to fixturing.

In the 89600 VSA, the Frequency Response measurement is implemented using the unbiased estimate equations shown in the previous two slides.

Other Multi-channel Measurements

Delay

Additive EVM

Rho (correlation coefficient)

Coherence

There are a number of other measurements that are possible when two coherent channels are available. Unfortunately there's not time to cover these in detail in this presentation.

Multi-Channel Measurements:
Delay

- **E911 and other geolocation services**
  - Many algorithms are based on time-difference of arrival techniques
  - Group delay through a device may change as a function of time, temperature, signal, control algorithms, etc.
- **Single channel techniques often limited by trigger accuracy, repeatability**
- **In self-adjusting DUT's, delay may be a function of signal statistics**
- **Multi-channel techniques require ADC's with a locked sample clocks**
- **Delay resolution is not limited by ADC sample interval**

As people start using time of arrival and time-difference of arrival techniques to locate cell phones, system timing parameters become much more important. For example, what is the delay through a multi-channel power amplifier with pre-distortion? More importantly, is it constant? As was previously shown, with two channels, very accurate delay measurements can be obtained with nearly infinite time resolution.

Multi-Channel Measurements:

EVM -- rms(e(t))

•Traditional EVM

- Requires demodulator to detect bits
- Bits used to synthesize a perfect, noise-free reference
- Assumes ideal signal at the input to the DUT
- Computed at symbol intervals

•Additive EVM

- Uses one input channel as a reference -- no demod required
- Does not assume ideal signal, or that the signal is a modulated carrier
- Reference channel measurement is not noise-free
- Computed over all time samples
- No measurement filter to limit signal bandwidth

This measurement is implemented in the Distortion Suite product mentioned earlier.

Multi-Channel Measurements:
Rho -- Correlation Coefficient

$$\rho_{xy} := \frac{\sigma_{xy}}{\sigma_x \cdot \sigma_y}$$

- **Single number metric with a magnitude between 0 and 1**
- **Traditional Rho**

  Requires demodulator to detect bits

  Bits used to synthesize a perfect, noise-free reference

  Assumes ideal signal at the input to the DUT

  Computed at chip intervals

- **General Case:**

  Uses one input channel as a reference -- no demod required

  Does not assume ideal signal, or that the signal is a modulated carrier

  Reference channel measurement is not noise-free

Rho, a normalized correlation coefficient is a measurement that existed well before the invention of CDMA phones.

Multi-Channel Measurements:
Coherence

$$\gamma_{xy}^{2}(f) := \frac{\left[\left| G_{xy}(f) \right|\right]^2}{G_{xx}(f) \cdot G_{yy}(f)}$$

- Similar in concept to Rho
- Coherence measures the percentage of the total output power that was linearly caused by the measured input signal verses frequency.
- Coherence function has a range of $0 \leq \gamma^2(f) \leq 1$.
- Coherence will be less than one when:
  - Noise is present
  - Non-linear input/output relationship
  - Spurious signals are injected between input and output

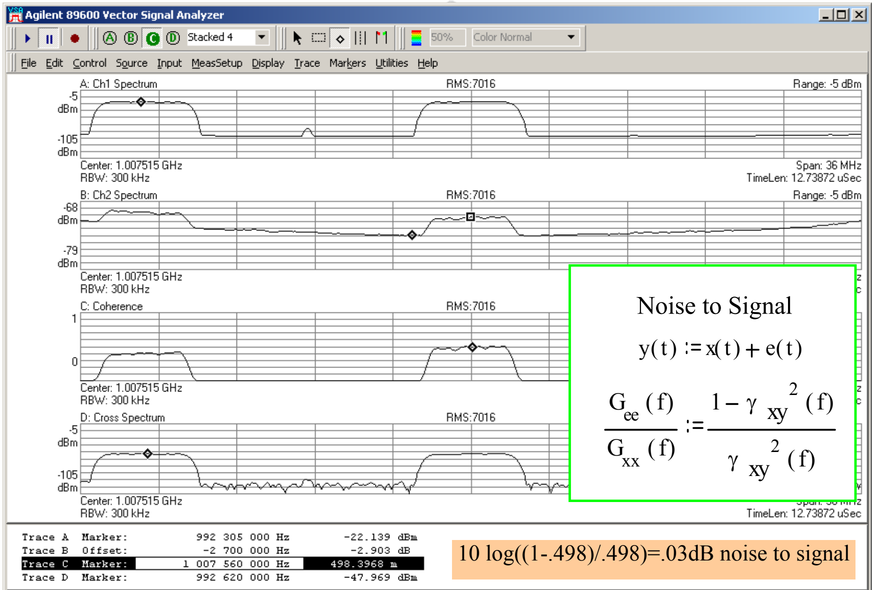
Signal to Noise

$$y(t) := x(t) + e(t)$$

$$\frac{G_{ee}(f)}{G_{xx}(f)} := \frac{1 - \gamma_{xy}^{2}(f)}{\gamma_{xy}^{2}(f)}$$

The coherence function is also an old and very useful measurement, but is likely to be new to those working in RF.

If you assume measured signals X(t) and y(t) with additive noise e(t), then the signal to noise can be estimated using the measured coherence.



In this measurement, a large signal with two active CDMA channels was connected to the first measurement channel. The same signal was also connected to the second channel, but was attenuated so that it's power-spectral-density (PSD) is the same as the instrument's noise floor. Since the signal PSD and noise PSD are the same, the spectrum has a 3dB "bump" where both signals are present.

The SNR on the first measurement channel is very good, so it can be assumed to be infinite. This allows the SNR equation shown to determine SNR (or it's inverse) to be determined. It's worth noting that this technique does not require the signal to be greater than the noise. This last point is important as this technique can be used to determine SNR of signals that exists well below some other signal.

Summary

•Vector Signal Analyzers with two phase-coherent, time-aligned measurements channels can be very useful in the design and test of Smart Antenna, MCPA, and other multi-port systems

•Previously, RF network measurements required network analyzers using sinusoidal signals to sweep amplitude and frequency. Many of the same measurements can now be made with any type of signal, including live signals.

•Measurement techniques exist that can actually remove noise and other unwanted signals from measurements.

Questions?